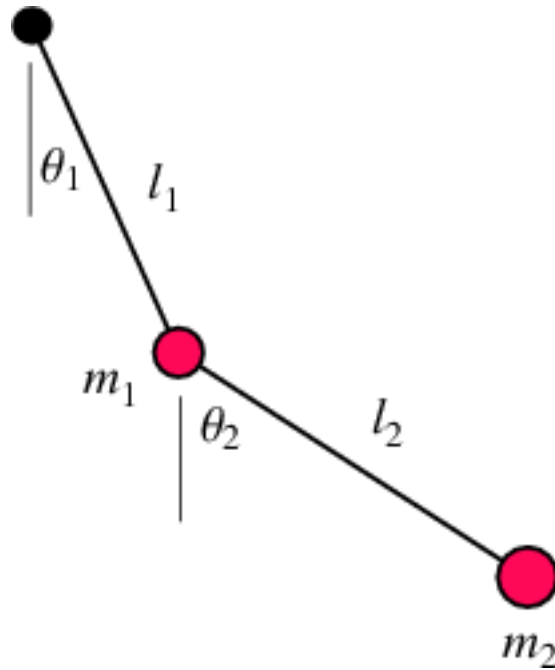


Double Pendulum and Chaos



The following derivation is from <http://scienceworld.wolfram.com/physics/DoublePendulum.html>

The equations of motion for the double pendulum are given by

$$\begin{aligned} & (m_1 + m_2)L_1 \frac{d^2\theta_1}{dt^2} + m_2L_2 \frac{d^2\theta_2}{dt^2} \cos(\theta_1 - \theta_2) \\ & + m_2L_2 \left(\frac{d\theta_2}{dt} \right)^2 \sin(\theta_1 - \theta_2) + g(m_1 + m_2) \sin(\theta_1) = 0 \\ & m_2L_2 \frac{d^2\theta_2}{dt^2} + m_2L_1 \frac{d^2\theta_1}{dt^2} \cos(\theta_1 - \theta_2) \\ & - m_2L_1 \left(\frac{d\theta_1}{dt} \right)^2 \sin(\theta_1 - \theta_2) + m_2g \sin(\theta_2) = 0 \end{aligned}$$

If we use the variables

$$\begin{aligned}u_1 &= \theta_1(t) \\u_2 &= \theta'_1(t) \\v_1 &= \theta_2(t) \\v_2 &= \theta'_2(t)\end{aligned}$$

we get the system

$$\begin{aligned}(m_1 + m_2)L_1 \frac{du_2}{dt} + m_2L_2 \frac{dv_2}{dt} \cos(u_1 - v_1) \\+ m_2L_2 (v_2)^2 \sin(u_1 - v_1) + g(m_1 + m_2) \sin(u_1) &= 0 \\m_2L_2 \frac{dv_2}{dt} + m_2L_1 \frac{du_2}{dt} \cos(u_1 - v_1) \\- m_2L_1 (u_2)^2 \sin(u_1 - v_1) + m_2g \sin(v_1) &= 0 \\ \frac{du_1}{dt} &= u_2(t) \\ \frac{dv_1}{dt} &= v_2(t)\end{aligned}$$

Using the substitutions $a = (m_1 + m_2)L_1$, $b = m_2L_2 \cos(u_1 - v_1)$, $c = m_2L_1 \cos(u_1 - v_1)$, $d = m_2L_2$, $e = -m_2L_2 (v_2)^2 \sin(u_1 - v_1) - g(m_1 + m_2) \sin(u_1)$, and $f = m_2L_1 (u_2)^2 \sin(u_1 - v_1) - m_2g \sin(v_1)$ we can write this system as

$$a \frac{du_2}{dt} + b \frac{dv_2}{dt} = e \tag{1}$$

$$c \frac{du_2}{dt} + d \frac{dv_2}{dt} = f \tag{2}$$

$$\frac{du_1}{dt} = u_2(t) \tag{3}$$

$$\frac{dv_1}{dt} = v_2(t) \tag{4}$$

Equations (1) and (2) can be solved for du_2/dt and dv_2/dt by

$$\frac{du_2}{dt} = \frac{ed - bf}{ad - cb} \quad (5)$$

$$\frac{dv_2}{dt} = \frac{af - ce}{ad - cb} \quad (6)$$

Equations (3)-(6) are now in the form that Matlab can use.

Initial Conditions The initial conditions given in the reference are (angles given in terms of radians)

$$u_1(0) = 1.5$$

$$u_2(0) = 0.0$$

$$v_1(0) = 3.0$$

$$v_2(0) = 0.0$$

The physical parameters are given by

$$L_1 = 1$$

$$L_2 = 2$$

$$m_1 = 2$$

$$m_2 = 1$$

$$g = 9.8$$

Matlab Project 3: Using the system of equations given above, and the parameters given above, reproduce the animated graphic contained in the reference.

Find the component plots $\theta_1(t) = u_1(t)$ vs t and $\theta_2(t) = v_1(t)$ vs t .

Examine several other initial conditions, and check for chaotic orbits.