Non Linear Autonomous Systems Analysis

We are given a nonlinear system of autonomous ordinary differential equations in the form

$$\frac{dx_1}{dt} = f_1(x_1, x_2)$$
$$\frac{dx_2}{dt} = f_2(x_1, x_2)$$

1. Find the critical points by looking for solutions of

$$f_1(x_1^*, x_2^*) = 0$$
$$f_2(x_1^*, x_2^*) = 0$$

2. Compute the jacobian of the system

$$J(x_1, x_2) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

3. Calculate the Jacobian matrix at **each** of the critical points found in 1. We will assume that the determinant of the Jacobian is non-zero at each of the critical points. This ensures that the critical points are **isolated**.

$$A = J(x_1^*, x_2^*)$$

4. Compute the eigenvalues of the linearized matrices A. This determines the nature of the phase plane in the neighborhood of the critical pionts.

5. Compute the eigenvectors of the linearized matrices A. This provides more detail about the shape of the trajectories near the critical points.

6. After determining the flow in the neighborhood of the critical points, finish filling in the phase plane with **separatrices**