

Group Project

1 Picard Iteration

```
> restart;  
> Picard := proc(t0,y0,f,x,y)  
> y0 + int(f(t,y(t)),t=t0..x)  
> end;
```

1.1 Part a) Linear Example:

```
> f := (x,y) -> y;  
f := (x, y) → y  
> y := x->1;  
y := 1  
> t0 := 0:  
> y0 := 1:  
> z := array(0..10);  
z := array(0..10, [])  
> z[0] := y;  
z0 := 1  
> z[1] := x -> Picard(t0,y0,f,x,z[0]):  
> z[1](x);  
1 + x  
> z[2] := x -> Picard(t0,y0,f,x,z[1]):  
> z[2](x);  
1 + x +  $\frac{1}{2}x^2$   
> z[3] := x -> Picard(t0,y0,f,x,z[2]):  
> z[3](x);  
1 + x +  $\frac{1}{2}x^2 + \frac{1}{6}x^3$   
> z[4] := x -> Picard(t0,y0,f,x,z[3]):  
> z[4](x);  
1 + x +  $\frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$ 
```

1.2 Part b) Nonlinear Example (with Unique Solution)

```
> f := (x,y) -> x-y^2;  
f := (x, y) → x - y2  
> y := x->0;  
y := 0  
> t0 := 0:  
> y0 := 0:  
> z := array(0..10);  
z := array(0..10, [])  
> z[0] := y;
```

```

> z0 := 0
> z[1] := x -> Picard(t0,y0,f,x,z[0]):
> z[1](x);

$$\frac{1}{2}x^2$$

> z[2] := x -> Picard(t0,y0,f,x,z[1]):
> z[2](x);

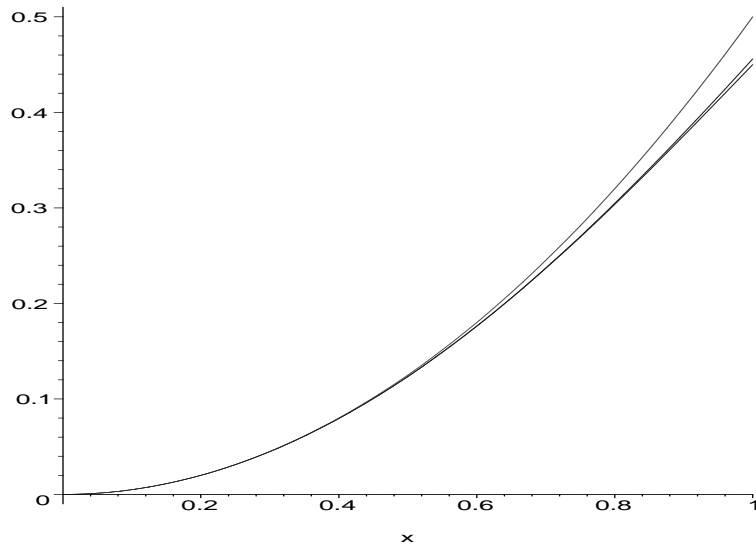
$$\frac{1}{2}x^2 - \frac{1}{20}x^5$$

> z[3] := x -> Picard(t0,y0,f,x,z[2]):
> z[3](x);

$$-\frac{1}{4400}x^{11} + \frac{1}{160}x^8 - \frac{1}{20}x^5 + \frac{1}{2}x^2$$

> plot({z[0](x),z[1](x),z[2](x),z[3](x)},x=0..1,color=[red,blue]);

```



1.3 Part c) Nonlinear Example (with Non Unique Solutions)

```

> f := (x,y) -> 3*y^(2/3);
<math display="block">f := (x, y) \rightarrow 3y^{(2/3)}y := x \rightarrow x - 2z := \text{array}(0..10, [])

```

```

      z0 := y
> z[1] := x -> Picard(t0,y0,f,x,z[0]):
> z[1](x);

```

$$\frac{9}{5}(x-2)^{(5/3)}$$

Must be a little more clever, so we do the calculations by hand

```

> z[2] := x -> Picard(t0,y0,f,x,z[1]):
> z[2](x);

```

Error, (in limit) invalid limiting point

```

> z[3] := x -> Picard(t0,y0,f,x,z[2]):
> z[3](x);

```

Error, (in limit) invalid limiting point

2 Analytical Solution of part b)

```

> restart:
> ode := diff(y(x),x) = x-y(x)^2;
      ode :=  $\frac{\partial}{\partial x} y(x) = x - y(x)^2$ 
> ic := y(0)=1;

```

$$ic := y(0) = 1$$

```

> dsolve({ode,ic},y(x));

```

$$y(x) = \frac{\frac{(-3\Gamma(\frac{2}{3})^2 3^{(2/3)} + 2\pi 3^{(5/6)}) \text{AiryAi}(1, x)}{3\Gamma(\frac{2}{3})^2 3^{(1/6)} + 2\pi 3^{(1/3)}} + \text{AiryBi}(1, x)}{\frac{(-3\Gamma(\frac{2}{3})^2 3^{(2/3)} + 2\pi 3^{(5/6)}) \text{AiryAi}(x)}{3\Gamma(\frac{2}{3})^2 3^{(1/6)} + 2\pi 3^{(1/3)}} + \text{AiryBi}(x)}$$

```

> simplify(%);

```

$$y(x) = \frac{-3 \text{AiryAi}(1, x) \Gamma(\frac{2}{3})^2 3^{(2/3)} - 2 \text{AiryAi}(1, x) \pi 3^{(5/6)} + 3 \text{AiryBi}(1, x) \Gamma(\frac{2}{3})^2 3^{(1/6)} + 2 \text{AiryBi}(1, x) \pi 3^{(1/3)}}{(-3 \text{AiryAi}(x) \Gamma(\frac{2}{3})^2 3^{(2/3)} + 2 \text{AiryAi}(x) \pi 3^{(5/6)} - 3 \text{AiryBi}(x) \Gamma(\frac{2}{3})^2 3^{(1/6)} - 2 \text{AiryBi}(x) \pi 3^{(1/3)})}$$

```

> evalf(%);

```

$$y(x) = -1. \frac{-4.25336217 \text{AiryAi}(1, x) + 15.66816282 \text{AiryBi}(1, x)}{4.25336217 \text{AiryAi}(x) - 15.66816282 \text{AiryBi}(x)}$$