Overview for Exam #3
[Sections 5.1-5.6,7.1-7.5,9.1-9.3]

- 5.2 Phase Plane (2 or more dimensions)

  Critical points of autonomous system

$$\frac{dx}{dt} = f_1(x, y)$$
$$\frac{dy}{dt} = f_2(x, y)$$

  are found by calculating the roots of $f_1(x, y) = 0 = f_2(x, y)$

  In two dimensions, the main types of behavior are

  1. Node ((stable or unstable)
  2. Spiral (stable or unstable)
  3. Saddle (unstable)

  In three dimensions, there are many other types of behavior possible (including chaotic)

- 5.3 Elimination Methods for constant coefficient equations

  Write the system in terms of $D = d/dx$, and then eliminate all but one of the equations (using algebraic methods). M equations of order N will be reduced to a signel $M \times N$ equation (usually).

- 5.4 Coupled Spring Mass Systems

  System of equations comes from force-balance laws.

  Normal modes come from complex exponential solutions of resulting equations (using elimination techniques of 5.3). Roots come in complex pairs.

- 5.5 Electical Circuits

  Khirchoff's two laws (conservation of charge/current and voltage drops) give system of equations of the same form as mass-spring system.

- 5.6 Numerical Methods for Higher Order Equations and Systems.

  Write the equation as a system of first order equations, and then apply one of the following methods:

  1. Euler
  2. Improved Euler
  3. Runge-Kutta

For simple equations, e.g.

$$\begin{aligned} y'' + y' + y &= \sin(t) \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned} \tag{1}$$

Euler or Runge-Kutta can be done by hand (i.e. by calculator).

- 7.2 Definitiion of Laplace Transform

  Find the Laplace Transform (of a continuous or piece-wise continuous function) by means of its definition:

  $$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) dt$$

  A table of Laplace transforms will be provided on the exam.

- 7.3 Properties of the Laplace Transform

  1. Linearity
  2. Action on Derivatives
  3. Multiplication by e^{at}
  4. Multiplication by t^n

- 7.4 Inverse Laplace Transform

  1. Apply transform to ode
  2. Solve for $Y(s)$
  3. Find inverse transform
     (a) Combine common terms
     (b) Put fractions into standard form
     (c) Use Method of Partial Fractions
     (d) Noramlize leading coefficient in denominator
     (e) Complete the square, $(s-a)^2 + b^2$
     (f) Normalize numerator, in terms of $s - a$
     (g) Table lookup (!)

- 7.5 Solving Initial Value Problems

  Incorporate initial conditions into $Y(s)$.

- 9.1 Matrix Methods

  Systems in Normal Form

- 9.2 Linear Algebraic Equations

  Gauss Jordan (row reduction) method

  Normal forms: upper triangular, diagonal

- 9.3 Matrices and Vectors

  Algebra of Matrices (addition, scalar multiplication, matrix/matrix multiplication). Non-commutativity of matrix-vector multiplication

  Find determinant.

  Find inverse by row-reduction.