

1. Given vectors $\vec{a} = \vec{i} - 2\vec{j}$, $\vec{b} = \langle -2, 3 \rangle$. Find

(a) a unit vector \vec{u} that has the same direction as $2\vec{b} + \vec{a}$.

(b) angle between \vec{a} and \vec{b}

(c) $\text{comp}_{\vec{b}}\vec{a}$, $\text{proj}_{\vec{b}}\vec{a}$.

2. Find the work done by a force of 20 lb acting in the direction $N50^\circ W$ in moving an object 4 ft due west.

3. Find the distance from the point $(-2,3)$ to the line $3x - 4y + 5 = 0$.

4. Find vector and parametric equations for the line passing through the points $A(1, -3)$ and $B(2, 1)$.

5. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2 + 1 & , \text{ if } x < 2, \\ x + 2 & , \text{ if } x \geq 2. \end{cases}$$

6. Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2 + 4}{3x^2 - 3}$.

7. Find $\frac{dy}{dx}$ for each function

(a) $y = (\sin x)^x$.

$$(b) \ y = \frac{\sqrt[5]{2x-1}(x^2-4)^2}{\sqrt[3]{1+3x}}$$

$$(c) \ y(t) = \sin^{-1} t, \ x(t) = \cos^{-1}(t^2).$$

$$(d) \ 2x^2 + 2xy + y^2 = x.$$

8. Find the equation of the tangent line to the curve $y = x\sqrt{5-x}$ at the point $(1,2)$.

9. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

(a) Find the velocity and acceleration functions.

(b) When is the particle moving upward?

(c) Find the distance that particle travels in the time interval $0 \leq t \leq 3$

10. The vector function $\vec{r}(t) = \langle t, 25t - 5t^2 \rangle$ represents the position of a particle at time t . Find the velocity, speed, and acceleration at $t = 1$.

11. Find y'' if $y = e^{-5x} \cos 3x$

12. Find $\frac{d^{50}}{dx^{50}} \cos 2x$

13. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

14. Find the quadratic approximation of $1/x$ for x near 4.

15. If $f(x) = x + x^2 + e^x$ and $g(x) = f^{-1}(x)$, find $g'(1)$.

16. Solve the equation $\ln(x + 6) + \ln(x - 3) = \ln 5 + \ln 2$

17. Find $\cos^{-1}\left(\sin \frac{5\pi}{4}\right)$.

18. Evaluate each limit:

(a) $\lim_{x \rightarrow 0} \frac{\sin x + \sin 2x}{\sin 3x}$

(b) $\lim_{x \rightarrow 0} (\cot x - \csc x)$

(c) $\lim_{x \rightarrow 0} x^{\sin x}$

19. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 2x^2 + x$ on $[-1,1]$.

20. A cup of coffee has a temperature of 200°F and is in a room that has a temperature of 70°F . After 10 min the temperature of the coffee is 150°F .
- (a) What is the temperature of the coffee after 15 min?
 - (b) When will the coffee have cooled to 100°F ?

21. For the function $y = x^2e^x$ find

- (a) All asymptotes.
- (b) Intervals on which the function is increasing/decreasing.
- (c) All local minima/local maxima.
- (d) Intervals on which the function is CU/CD.
- (e) Inflection points.

22. A cylindrical can without a top is made to contain $V \text{ cm}^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

23. Find the derivative of the function $f(x) = \int_0^{\sqrt{x}} \frac{t^2}{t^2 + 1} dt$

24. Evaluate the integral:

$$(a) \int_1^2 \left(x + \frac{1}{x}\right)^2 dx$$

$$(b) \int_1^2 \frac{x^2 + 1}{\sqrt{x}} dx$$

$$(c) \int_0^{\pi/2} (\cos t + 2 \sin t) dt$$

25. Find the area under the curve $y = x^2 + 3x - 2$ from 1 to 4. Use equal subintervals and take x_i^* to be the right end-point of the i -th interval

26. Express the limit $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \frac{1}{1 + (i/n)^2}$ as a definite integral. Do not evaluate it.

27. A particle moves in a straight line and has acceleration given by $a(t) = t^2 - t$. Its initial velocity is $v(0) = 2$ cm/s and its initial displacement is $s(0) = 1$ cm. Find the position function $s(t)$.

28. Find the vector function $\vec{r}(t)$ that gives the position of a particle at time t having the acceleration $\vec{a}(t) = 2t\vec{i} + \vec{j}$, initial velocity $\vec{v}(0) = \vec{i} - \vec{j}$, and initial position $(1, 0)$.