Math151 **Sample Problems for the Final** Fall 2016

1. Given vectors $\vec{a} = \vec{i} - 2\vec{j}$, $\vec{b} = < -2, 3 >$. Find

(a) a unit vector $\vec{u}$ that has the same direction as $2\vec{b} + \vec{a}$ $= 2<-2,3> + <1,-2>$

$= <-4,6> + <1,-2> = <-3,4>$

$|2\vec{b}+\vec{a}| = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\vec{u} = \dfrac{2\vec{b}+\vec{a}}{|2\vec{b}+\vec{a}|} = \dfrac{<-3,4>}{5} = \boxed{<-\frac{3}{5}, \frac{4}{5}>}$

(b) angle between $\vec{a}$ and $\vec{b}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{<1,-2>\cdot<-2,3>}{\sqrt{1^2+(-2)^2}\cdot\sqrt{(-2)^2+3^2}} = \dfrac{-2-6}{\sqrt{5}\sqrt{13}} = -\dfrac{8}{\sqrt{65}}$

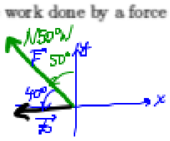
$\theta = \cos^{-1}\left(-\frac{8}{\sqrt{65}}\right) = \boxed{\pi - \cos^{-1}\left(\frac{8}{\sqrt{65}}\right)}$

(c) $\text{comp}_{\vec{b}}\vec{a}$, $\text{proj}_{\vec{b}}\vec{a}$.

$\text{comp}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \boxed{\dfrac{-8}{\sqrt{13}}}$

$\text{proj}_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = -\dfrac{8}{13}<-2,3> = \boxed{<\frac{16}{13}, -\frac{24}{13}>}$

2. Find the work done by a force of 20 lb acting in the direction N50°W in moving an object 4 ft due west.



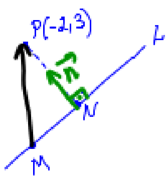
$|\vec{F}| = 20$, $|\vec{D}| = 4$

$W = \vec{F}\cdot\vec{D} = |\vec{F}|\cdot|\vec{D}|\cos\theta$

$= (20)(4)\cos 40°$

$= \boxed{80\cos 40°}$

3. Find the distance from the point (-2,3) to the line $3x - 4y + 5 = 0$.



Point on $3x-4y+5=0$

$x=0$, $y=\frac{5}{4}$

$M(0, \frac{5}{4})$

$D = |\vec{PN}| = |\text{comp}_{\vec{n}}\vec{MP}|$

$\vec{n} = <3,-4>$

$\vec{MP} = <0-(-2), \frac{5}{4} - 3>$

$= <2, -\frac{7}{4}>$

$D = \left|\text{comp}_{\vec{n}}\vec{MP}\right| = \left|\dfrac{\vec{n}\cdot\vec{MP}}{|\vec{n}|}\right| = \left|\dfrac{<3,-4>\cdot<2,-\frac{7}{4}>}{\sqrt{3^2+(-4)^2}}\right|$

$= \left|\dfrac{3(2)+4\cdot\frac{7}{4}}{\sqrt{9+16}}\right| = \boxed{\dfrac{13}{5}}$

Distance from $P(x_1,y_1)$ to the line $Ax+By+C=0$ is

$\boxed{d = \left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|}$ $d = \left|\dfrac{3(-2)-4(3)+5}{\sqrt{9+16}}\right| = \boxed{\dfrac{13}{5}}$

4. Find vector and parametric equations for the line passing through the points $A(1,-3)$ and $B(2,1)$.

the line is parallel to $\vec{AB} = \langle 2-1, 1-(-3)\rangle = \langle 1, 4\rangle$

Vector equation: $\vec{r}(t) = \langle 1,-3\rangle + t\langle 1,4\rangle$ or $\vec{r}(t) = \langle 2,1\rangle + t\langle 1,4\rangle$

parametric equations: $\begin{cases} x(t) = 1+t \\ y(t) = -3+4t \end{cases}$ $\begin{cases} x(t) = 2+t \\ y(t) = 1+4t \end{cases}$

5. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2+1 & , \text{ if } x < 2, \\ x+2 & , \text{ if } x \geq 2. \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (x^2+1) = 2^2+1 = 5$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} (x+2) = 2+2 = 4$

$\Rightarrow \lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

$\boxed{f(x) \text{ has a jump discontinuity @ } x=2}$

6. Find the vertical and horizontal asymptotes of the curve $y = \dfrac{(x^2+4)(x+1)}{(3x^2-3)(x+2)} = \dfrac{(x^2+4)(x+1)}{3(x^2-1)(x+2)} = \dfrac{(x^2+4)\cancel{(x+1)}}{3\cancel{(x+1)}(x-1)(x+2)}$

Vertical asymptotes: $\boxed{x=1}$ $\boxed{x=-2}$ | removable discontinuity @ $x=-1$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{x^2+4}{3(x-1)(x+2)} = \dfrac{1}{3}$

Horizontal asymptote $\boxed{y=\frac{1}{3}}$

du

7. Find $\frac{dy}{dx}$ for each function

(a) $y = (\sin x)^x$.

Logarithmic differentiation.

$\ln y = \ln(\sin x)^x$

$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln(\sin x)$

$\frac{y'}{y} = \ln(\sin x) + x \frac{1}{\sin x} \cos x$

$\frac{y'}{y} = \ln(\sin x) + x \cot x$

$y' = (\ln(\sin x) + x \cot x) y$

$\boxed{y' = (\sin x)^x (\ln(\sin x) + x \cot x)}$

(b) $y = \dfrac{\sqrt[5]{2x-1}(x^2-4)^2}{\sqrt[3]{1+3x}}$ Logarithmic differentiation.

$$\ln y = \ln \frac{\sqrt[5]{2x-1}\,(x^2-4)^2}{\sqrt[3]{1+3x}}$$

$$= \ln\left(\sqrt[5]{2x-1}\right) + \ln(x^2-4)^2 - \ln\left(\sqrt[3]{1+3x}\right)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\left(\frac{1}{5}\ln(2x-1) + 2\ln(x^2-4) - \frac{1}{3}\ln(1+3x)\right)$$

$$\frac{y'}{y} = \frac{1}{5}\cdot\frac{2}{2x-1} + 2\cdot\frac{2x}{x^2-4} - \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{1+3x}$$

$$y' = y\left(\frac{2}{5(2x-1)} + \frac{4x}{x^2-4} - \frac{1}{1+3x}\right)$$

$$y' = \boxed{\frac{\sqrt[5]{2x-1}\,(x^2-4)^2}{\sqrt[3]{1+3x}}\left(\frac{2}{5(2x-1)} + \frac{4x}{x^2-4} - \frac{1}{1+3x}\right)}$$

(c) $y(t) = \sin^{-1} t$, $x(t) = \cos^{-1}(t^2)$.

$$(\sin^{-1}(t))' = \frac{1}{\sqrt{1-t^2}}, \quad (\cos^{-1}(t))' = -\frac{1}{\sqrt{1-t^2}}$$

$$(1-t^4) = (1-t^2)(1+t^2)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{(\sin^{-1} t)'}{(\cos^{-1}(t^2))'} = \frac{\frac{1}{\sqrt{1-t^2}}}{-\frac{1}{\sqrt{1-(t^2)^2}}\,(t^2)'} = \frac{\frac{1}{\sqrt{1-t^2}}}{-\frac{2t}{\sqrt{1-t^4}}} = -\frac{\sqrt{1-t^4}}{2t\sqrt{1-t^2}}$$

$$= \boxed{-\frac{\sqrt{1+t^2}}{2t}}$$

(d) $\frac{d}{dx}\left(2x^2 + 2xy + y^2\right) = \frac{d}{dx}x$.

$$4x + \underbrace{2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx}}_{} = 1$$

$$\frac{dy}{dx}(2x+2y) = 1 - 4x - 2y$$

$$\frac{dy}{dx} = \boxed{\frac{1-4x-2y}{2x+2y}}$$

8. Find the equation of the tangent line to the curve $y = x\sqrt{5-x}$ at the point $(\underbrace{1}_{a},\underbrace{2}_{f(a)})$.

$y = f'(a)(x-a) + f(a)$ – tangent line to $y = f(x)$ at $x = a$.

$y' = \sqrt{5-x} + x\,\frac{1}{2}(5-x)^{-1/2}(5-x)'$

$= \sqrt{5-x} - \frac{x}{2}(5-x)^{-1/2}$

$y'(1) = \sqrt{5-1} - \frac{1}{2}(5-1)^{-1/2} = 2 - \frac{1}{2\cdot(2)} = 2 - \frac{1}{4} = \frac{7}{4}$

$\boxed{y = \frac{7}{4}(x-1) + 2}$

9. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

(a) Find the velocity and acceleration functions.

$v(t) = y'(t) = 3t^2 - 12$

$a(t) = 6t$

(b) When is the particle moving upward? $v(t) > 0$

$3t^2 - 12 > 0$

$t^2 - 4 > 0$

$(t-2)(t+2) > 0$

$+ \quad -2 \quad - \quad 0 \quad - \quad 2 \quad +$

moves up when $\boxed{t > 2}$

(c) Find the distance that particle travels in the time interval $0 \leq t \leq 3$

$\cancel{D} = |s(2) - s(0)| + |s(3) - s(2)|$

$= |2^3 - 12(2) + 3 - 3| + |3^3 - 12(3) + 3 - 2^3 + 12(2) - 3|$

$= |8 - 24| + |27 - 36 - 8 + 24| = 16 + 7 = \boxed{23}$

10. The vector function $\vec{r}(t) = < t, 25t - 5t^2 >$ represents the position of a particle at time t. Find the velocity, speed, and acceleration at $t = 1$.

$\vec{v}(t) = \vec{r}\,'(t) = <1, 25 - 10t>$, $\boxed{\vec{v}(1) = <1, 15>}$

$s(1) = |\vec{v}(1)| = \sqrt{1 + 15^2} = \boxed{\sqrt{226}}$

$\vec{a}(t) = \vec{v}\,'(t) = <0, -10>$, $\boxed{\vec{a}(1) = <0, -10>}$

11. Find y'' if $y = e^{-5x}\cos 3x$

$$y' = -5e^{-5x}\cos 3x + e^{-5x}(-\sin 3x)(3)$$

$$y'' = 25e^{-5x}\cos 3x - 5e^{-5x}(-\sin 3x)(3) + (-5)e^{-5x}(-\sin 3x)(3) - 3(3)e^{-5x}\cos 3x$$

$$= 25e^{-5x}\cos 3x + 30e^{-5x}\sin 3x - 9e^{-5x}\cos 3x$$

$$\boxed{16e^{-5x}\cos 3x + 30e^{-5x}\sin 3x}$$

12. Find $\dfrac{d^{50}}{dx^{50}}\cos 2x$

$f(x) = \cos 2x$

$f'(x) = -2\sin 2x$

$f''(x) = -4\cos 2x$

$f'''(x) = 8\sin 2x$

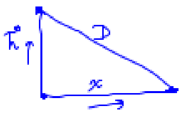
$f^{IV}(x) = 16\cos 2x$

$50 = 48+2 \Rightarrow$ remainder is $\underline{2}$.

$f^{(50)}(x)$ is similar to $f''(x)$

$$\boxed{f^{(50)}(x) = -2^{50}\cos 2x}$$

13. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?



$\dfrac{dx}{dt} = 15, \quad \dfrac{dh}{dt} = 5$

Find $\dfrac{dD}{dt}$ when $t = 3$

$\dfrac{d}{dt} D^2 = \dfrac{d}{dt}(x^2 + h^2)$

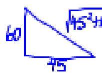
$\cancel{2}D\dfrac{dD}{dt} = \cancel{2}x\dfrac{dx}{dt} + \cancel{2}h\dfrac{dh}{dt}$

$\dfrac{dD}{dt} = \dfrac{1}{D}\left(x\dfrac{dx}{dt} + h\dfrac{dh}{dt}\right)$

after 3 sec

$x = 3(15) = 45$

$h = 45 + 3(5) = 60$



$\sqrt{45^2+60^2} = 75 = D$

$\dfrac{dD}{dt} = \dfrac{1}{75}\left(45(15) + 60(5)\right) = \dfrac{1}{\cancel{15}\cdot 5}\left(\cancel{15}\cdot 45 + \cancel{15}\cdot 4(5)\right)$

$= \dfrac{65}{5} = \boxed{13 \text{ ft/s}}$

14. Find the quadratic approximation of $1/x$ for x near 4.

$Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$f(x) = \frac{1}{x}$	$f(4) = \frac{1}{4}$
$f'(x) = -\frac{1}{x^2}$	$f'(4) = -\frac{1}{16}$
$f''(x) = \frac{2}{x^3}$	$f''(4) = \frac{2}{64} = \frac{1}{32}$

$\frac{1}{x} \approx f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2$

$= \boxed{\frac{1}{4} - \frac{1}{16}(x-4) + \frac{1}{64}(x-4)^2}$

$f'(x) = 2x + 1 + e^x$

15. If $f(x) = x + x^2 + e^x$ and $g(x) = f^{-1}(x)$, find $g'(1)$.

$g'(1) = \frac{1}{f'(g(1))}$, $g(1) = x \Leftrightarrow f(x) = 1$

$f(x) = x + x^2 + e^x$

$x = 0:\quad 0 + 0 + e^0 = 1$

$f(0) = 1 \Leftrightarrow g(1) = 0$

$g'(1) = \frac{1}{f'(0)} = \frac{1}{2(0) + 1 + e^0} = \boxed{\frac{1}{2}}$

16. Solve the equation $\ln(x+6) + \ln(x-3) = \ln 5 + \ln 2$

Domain: $x + 6 > 0 \Rightarrow x > -6$

$x - 3 > 0 \Rightarrow x > 3$

$\boxed{x > 3}$

$\ln(x+6)(x-3) = \ln 10$

$(x+6)(x-3) = 10$

$x^2 + 3x - 18 = 10$

$x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0$

$x_1 = -7$, $\boxed{x_2 = 4}$

not in the domain

17. Find $\cos^{-1}\left(\sin\frac{5\pi}{4}\right)$. $= \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

18. Evaluate each limit:

(a) $\lim_{x\to 0} \frac{\sin x + \sin 2x}{\sin 3x}$ $= \left| \frac{0}{0} \right| = \lim_{x\to 0} \frac{\cos x + 2\cos 2x}{3\cos 3x} = \frac{1+2}{3} = \boxed{1}$

(b) $\lim_{x\to 0}(\cot x - \csc x)$ $= \lim_{x\to 0}\left(\frac{\cos x}{\sin x} - \frac{1}{\sin x}\right) = \lim_{x\to 0} \frac{\cos x - 1}{\sin x} \quad \left| \frac{0}{0} \right|$

$= \lim_{x\to 0} \frac{-\sin x}{\cos x} = \boxed{0}$

(c) $\lim_{x\to 0} x^{\sin x} = \lim_{x\to 0} e^{\sin x \ln x} = e^{\lim_{x\to 0} \sin x \ln x}$

$$\lim_{x\to 0} \sin x \ln x = |0.\infty| = \lim_{x\to 0} \frac{\ln x}{\csc x} = \left|\frac{\infty}{\infty}\right| = \lim_{x\to 0} \frac{\frac{1}{x}}{-\csc x \cot x}$$

$$= -\lim_{x\to 0} \frac{\sin x \tan x}{x} = -\lim_{x\to 0} \cancel{\frac{\sin x}{x}}^{1} \cdot \tan x = -\lim_{x\to 0} \tan x = 0$$

$$\boxed{\lim_{x\to 0} x^{\sin x} = e^{0} = 1}$$

19. Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 2x^2 + x$ on $[-1,1]$.

Critical numbers: $f'(x) = 3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x_1 = \frac{1}{3}, \quad x_2 = 1$

$f(-1) = -1-2-1 = \boxed{-4 \text{ abs min value}}$

$f\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} = \boxed{\frac{4}{27} \text{ abs max value}}$

$f(1) = 1-2+1 = 0$

20. A cup of coffee has a temperature of 200°F and is in a room that has a temperature of 70°F. After 10 min the temperature of the coffee is 150°F.

(a) What is the temperature of the coffee after 15 min?

(b) When will the coffee have cooled to 100°F?

$T(t)$ is the temperature of the coffee after t min.

$$\frac{dT}{dt} = k(T-70), \quad T(0)=200, \quad T(10)=150$$

substitution: $\frac{d}{dt}u(t) = \frac{d}{dt}\left(T(t)-70\right)$, $u(0) = T(0) - 70 = 200 - 70 = 130$

$$\frac{du}{dt} = \frac{dT}{dt} - 0$$

$$u: \quad \frac{du}{dt} = ku, \quad u(0) = 130$$

$$T(t) - 70 = u(t) = u(0)e^{kt} = 130e^{kt}$$

$$T(t) = 70 + 130e^{kt}$$

$$T(10) = 70 + 130e^{10k} = 150$$

$$130e^{10k} = 80 \Rightarrow e^{10k} = \frac{8}{13} \Rightarrow 10k = \ln\frac{8}{13} \Rightarrow k = \frac{1}{10}\ln\frac{8}{13}$$

$$\boxed{T(t) = 70 + 130e^{\frac{t}{10}\ln\frac{8}{13}}}$$

(a) $T(15) = 70 + 130e^{\frac{15}{10}\ln\frac{8}{13}} = \boxed{70 + 130e^{\frac{3}{2}\ln\frac{8}{13}}}$

(b) Find t such that $T(t) = 100$

$$T(t) = 70 + 130e^{\frac{t}{10}\ln\frac{8}{13}} = 100$$

$$130e^{\frac{t}{10}\ln\frac{8}{13}} = 30$$

$$e^{\frac{t}{10}\ln\frac{8}{13}} = \frac{3}{13}$$

$$\frac{t}{10}\ln\frac{8}{13} = \ln\frac{3}{13} \Rightarrow t = \boxed{\frac{10\ln\frac{3}{13}}{\ln\frac{8}{13}}}$$

21. For the function $y = x^2e^x$ find

(a) All asymptotes.

(b) Intervals on which the function is increasing/decreasing.

(c) All local minima/local maxima.

(d) Intervals on which the function is CU/CD.

(e) Inflection points.

(a) $\lim\limits_{x\to\infty} x^2e^x = \infty$

$$\lim_{x\to-\infty} x^2e^x = |0\cdot\infty| = \lim_{x\to-\infty}\frac{x^2}{e^{-x}} = \left|\frac{\infty}{\infty}\right| = \lim_{x\to-\infty}\frac{2x}{-e^{-x}}\left|\frac{\infty}{\infty}\right|$$

$$= \lim_{x\to-\infty}\frac{2}{e^{-x}} = 0 \Rightarrow \boxed{y=0 \text{ is the horizontal asymptote when } x\to-\infty}$$

(b) $y' = 2xe^x + x^2e^x = e^x(2x+x^2) > 0$

$e^x > 0$ for all $x \Rightarrow$ $2x+x^2>0$

$x(x+2)>0$

$f(x)$

$f'(x)$: $+$ | -2 | $-$ | 0 | $+$

-2 local max

0 local min

$$\boxed{\begin{array}{l} f(x) \text{ is increasing on } (-\infty,-2)\cup(0,\infty) \\ f(x) \text{ is decreasing on } (-2,0) \end{array}}$$

(c)
$$\boxed{\begin{array}{l} f \text{ has a local max @ } (-2, 4e^{-2}) \\ f \text{ has a local min @ } (0, 0) \end{array}}$$

(d) $y'' = 2e^x + 4xe^x + x^2e^x = e^x(2+4x+x^2) > 0 \Rightarrow$

$2+4x+x^2 = 0$

$x_1 = \dfrac{-4+\sqrt{8}}{2} = -2+\sqrt{2}$

$x_2 = \dfrac{-4-\sqrt{8}}{2} = -2-\sqrt{2}$

$2+4x+x^2>0$

$f(x)$: $\cup$ | $\cap$ | $\cup$

$f''(x)$: $+$ | $-2-\sqrt{2}$ | $-$ | $-2+\sqrt{2}$ | $+$

$$\boxed{\begin{array}{l} f \text{ is CU on } (-\infty,-2-\sqrt{2})\cup(-2+\sqrt{2},\infty) \\ f \text{ is CD on } (-2-\sqrt{2},-2+\sqrt{2}) \end{array}}$$

(e) f has $\boxed{\text{inflection points @ } x=-2-\sqrt{2} \text{ and } x=-2+\sqrt{2}}$

22. A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

$V = \pi r^2 h$, minimize the surface area $A = \pi r^2 + 2\pi r h$

$h = \frac{V}{\pi r^2}$

$A = \pi r^2 + 2\pi r \cdot \frac{V}{\pi r^2} = \pi r^2 + \frac{2V}{r}$

$A' = 2\pi r - \frac{2V}{r^2} = 0$

$\pi r = \frac{V}{r^2} \implies r^3 = \frac{V}{\pi} \implies \boxed{r = \sqrt[3]{\frac{V}{\pi}}}$

$h = \frac{V}{\pi r^2} = \frac{V}{\pi \sqrt[3]{\frac{V}{\pi}}} = \boxed{\sqrt[3]{\frac{V}{\pi}} = h}$

$A'' = 2\pi + \frac{2V}{r^3}$, $A''\left(\sqrt[3]{\frac{V}{\pi}}\right) = 3\pi > 0$, so A has a minimum when $r = \sqrt[3]{\frac{V}{\pi}}$

23. Find the derivative of the function $f(x) = \int_0^{\sqrt{x}} \frac{t^2}{t^2+1}\,dt$

$u = \sqrt{x}$

$f'(x) = \frac{u^2}{u^2+1}\, u'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^2+1}\, \frac{1}{2\sqrt{x}} = \frac{x}{x+1} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{\sqrt{x}}{2(x+1)}}$

10

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24. Evaluate the integral:

(a) $\int_1^2 \left(x+\frac{1}{x}\right)^2 dx = \int_1^2 \left(x^2 + 2x\frac{1}{x} + \frac{1}{x^2}\right)dx = \int_1^2 \left(x^2 + 2 + \frac{1}{x^2}\right)dx$

$= \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]_1^2 = \frac{8}{3} + 4 - \frac{1}{2} - \left(\frac{1}{3} + 2 - 1\right) = \boxed{\frac{29}{6}}$

(b) $\int_1^2 \frac{x^2+1}{\sqrt{x}}dx = \int_1^2 (x^2+1)\, x^{-1/2}\, dx = \int_1^2 \left(x^{2-1/2} + x^{-1/2}\right)dx = \int_1^2 \left(x^{3/2} + x^{-1/2}\right)dx$

$= \left[\frac{x^{3/2+1}}{3/2+1} + \frac{x^{-1/2+1}}{-1/2+1}\right]_1^2 = \left[\frac{2}{5}x^{5/2} + 2x^{1/2}\right]_1^2 = \frac{2}{5}2^{5/2} + 2\sqrt{2} - \frac{2}{5} - 2$

$= \frac{2}{5}(4)\sqrt{2} + 2\sqrt{2} - \frac{12}{5} = \boxed{\frac{18}{5}\sqrt{2} - \frac{12}{5}}$

(c) $\int_0^{\pi/2} (\cos t + 2\sin t)dt = \left[\sin t - 2\cos t\right]_0^{\pi/2}$

$= \sin\frac{\pi}{2} - 2\cos\frac{\pi}{2} - \sin 0 + 2\cos 0$

$= 1 + 2 = \boxed{3}$

25. Find the area under the curve $y = x^2 + 3x - 2$ from 1 to 4. Use equal subintervals and take x_i^* to be the right end-point of the i-th interval

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$

Partition points.

$x_0 = 1$

$x_1 = 1 + \Delta x = 1 + \frac{3}{n}$

$x_2 = x_1 + \Delta x = 1 + 2\Delta x = 1 + 2 \cdot \frac{3}{n}$

$\ldots$

$x_i = 1 + i \cdot \frac{3}{n} = x_i^*$

$\ldots$

$x_n = 4$

$f(x_i^*) = \left(1 + \frac{3i}{n}\right)^2 + 3\left(1 + \frac{3i}{n}\right) - 2$

$= 1 + \frac{6i}{n} + \frac{9i^2}{n^2} + 3 + \frac{9i}{n} - 2$

$= \frac{9i^2}{n^2} + \frac{15i}{n} + 2$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2} + \frac{15i}{n} + 2\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9}{n^2}\underbrace{\sum_{i=1}^{n} i^2}_{\frac{n(n+1)(2n+1)}{6}} + \frac{15}{n}\underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + 2\underbrace{\sum_{i=1}^{n} 1}_{n}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{\cancel{9}\,3}{\cancel{n^2}}\,\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,2} + \frac{15}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{2} + 2n\right)$$

$$= \lim_{n\to\infty}\left(\frac{9}{2}\,\frac{(n+1)(2n+1)}{n^2} + \frac{45}{2}\,\frac{(n+1)}{n} + \frac{6\cancel{n}}{\cancel{n}}\right) = \frac{9}{2}(2) + \frac{45}{2} + 6$$

$$= 37.5 = \boxed{\frac{75}{2}}$$

26. Express the limit $\lim_{n\to\infty} \overbrace{\frac{1}{n}}^{\Delta x} \sum_{i=1}^{n} \frac{1}{1+(i/n)^2}$ as a definite integral. Do not evaluate it.

$\frac{1}{1+(\frac{i}{n})^2} = f(x_i)$, $x_i = \frac{i}{n}$ $\Rightarrow$ $f(x) = \frac{1}{1+x^2}$

$x_i = \frac{i}{n}$ $\Rightarrow$ $i = 0$, $x_0 = 0 = a$

$i = n$, $x_n = 1 = b$

$\lim = \boxed{\int_0^1 \frac{1}{1+x^2}\,dx}$

27. A particle moves in a straight line and has acceleration given by $a(t) = t^2 - t$. Its initial velocity is $v(0) = 2$ cm/s and its initial displacement is $s(0) = 1$ cm. Find the position function $s(t)$.

$v(t) = \int (t^2 - t)\,dt = \frac{t^3}{3} - \frac{t^2}{2} + C$, $v(0) = C = 2$

$v(t) = \frac{t^3}{3} - \frac{t^2}{2} + 2$

$s(t) = \int \left(\frac{t^3}{3} - \frac{t^2}{2} + 2\right) dt = \frac{t^4}{12} - \frac{t^3}{6} + 2t + C$

$s(0) = C = 1$

$\boxed{s(t) = \frac{t^4}{12} - \frac{t^3}{6} + 2t + 1}$

28. Find the vector function $\vec{r}(t)$ that gives the position of a particle at time t having the acceleration $\vec{a}(t) = 2t\vec{\imath} + \vec{\jmath}$, initial velocity $\vec{v}(0) = \vec{\imath} - \vec{\jmath}$, and initial position $(1, 0)$.

$\vec{v}(t) = \langle \int 2t\,dt, \int 1\,dt \rangle$

$= \langle t^2 + C_1, t + C_2 \rangle$, $v(\vec{0}) = \langle C_1, C_2 \rangle = \langle 1, -1 \rangle$

$C_1 = 1$, $C_2 = -1$

$\vec{v}(t) = \langle t^2 + 1, t - 1 \rangle$

$\vec{r}(t) = \langle \int (t^2+1)\,dt, \int (t-1)\,dt \rangle$

$= \langle \frac{t^3}{3} + t + C_3, \frac{t^2}{2} - t + C_4 \rangle$

$\vec{r}(0) = \langle C_3, C_4 \rangle = \langle 1, 0 \rangle \Rightarrow C_3 = 1$, $C_4 = 0$

$\boxed{\vec{r}(t) = \langle \frac{t^3}{3} + t + 1, \frac{t^2}{2} - t \rangle}$