$0 \le x \le \frac{\pi}{2}$

$\sin(\pi + x) = -\sin x$

1. Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$.

$\frac{17\pi}{12} = \pi + \frac{5\pi}{12}$; $\frac{17\pi}{12}$ is in the 3rd quadrant

$\pi < \frac{17\pi}{12} < \frac{3\pi(6)}{2(6)} = \frac{16\pi}{12}$.

$\sin\left(\frac{17\pi}{12}\right) = -\sin\left(\frac{5\pi}{12}\right)$

Convert $\frac{5\pi}{12} = \frac{5\cancel{\pi}}{\cancel{2}\cancel{12}}\left(\frac{\cancel{180}^{30}}{\cancel{\pi}}\right)^{\circ} = \frac{150^{\circ}}{2} = 75^{\circ} = 45^{\circ} + 30^{\circ}$

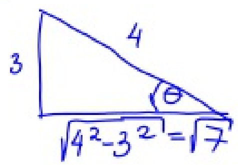
$-\sin 75^{\circ} = -\sin(45^{\circ} + 30^{\circ}) = -\left(\sin 45^{\circ}\cos 30^{\circ} + \cos 45^{\circ}\sin 30^{\circ}\right)$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= -\left(\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}\right) = \boxed{-\frac{\sqrt{2}}{4}(\sqrt{3}+1)}$

2. If $\csc\theta = -\frac{4}{3}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, find $\overset{+}{\cos\theta}$, $\overset{-}{\sin\theta}$, $\overset{-}{\tan\theta}$, $\overset{-}{\cot\theta}$.

4th quadrant



$\csc\theta = \frac{1}{\sin\theta} = -\frac{4}{3} \Rightarrow \boxed{\sin\theta = -\frac{3}{4}}$

$\boxed{\cos\theta = \frac{\sqrt{7}}{4}}$ $\boxed{\tan\theta = -\frac{3}{\sqrt{7}}}$

$\boxed{\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{7}}{3}}$

3. Prove the following:

$(a-b)(a+b)=a^2-b^2$

(a) $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$

RHS: $\sin(x+y)\sin(x-y) = [\underbrace{\sin x\cos y}_{a} + \underbrace{\cos x \sin y}_{b}](\sin x\cos y - \cos x\sin y)$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

$\cos^2 x = 1-\sin^2 x$

$= (\sin x\cos y)^2 - (\cos x\sin y)^2$

$= \sin^2 x\underbrace{\cos^2 y}_{1-\sin^2 y} - \underbrace{\cos^2 x}_{1-\sin^2 x}\sin^2 y$

$= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y$

$= \sin^2 x - \cancel{\sin^2 x\sin^2 y} - \sin^2 y + \cancel{\sin^2 x\sin^2 y}$

$= \sin^2 x - \sin^2 y = LHS$

(b) $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = 2\sec^2 x$

LHS: $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = \dfrac{1+\cancel{\sin x}+1-\cancel{\sin x}}{(\underbrace{1}_{a}-\underbrace{\sin x}_{b})(1+\sin x)} = \dfrac{2}{\underbrace{1-\sin^2 x}_{\cos^2 x}} = \dfrac{2}{\cos^2 x} = 2\sec^2 x$

$(a-b)(a+b)=a^2-b^2$

4. Solve the equation, if $0 \le x \le 2\pi$

(a) $\frac{2\sin^2 x}{2} = \frac{1}{2}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{1}{\sqrt{2}}$ or $\sin x = -\frac{1}{\sqrt{2}}$

$x = \boxed{\frac{\pi}{4}, \frac{3\pi}{4}}$

$x = \boxed{\frac{5\pi}{4}, \frac{7\pi}{4}}$

(b) $2 + \cos \underline{2x} = 3\cos \underline{x}$

$\cos 2x = \cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x} = \cos^2 x - (1-\cos^2 x)$

$\cos 2x = 2\cos^2 x - 1$

$\sin 2x = 2\sin x \cos x$

$2 + (2\cos^2 x - 1) = 3\cos x$

$2\cos^2 x - 3\cos x + 1 = 0$

substitution: $\cos x = t$, $0 \le t \le 1$

$2t^2 - 3t + 1 = 0$

$t_1 = \frac{3 + \sqrt{3^2 - 2(4)}}{4} = \frac{3+1}{4} = 1$

$t_2 = \frac{3-1}{4} = \frac{1}{2}$

$\cos x = 1$

$x = \boxed{0, 2\pi}$

$\cos x = \frac{1}{2}$

$x = \boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

5. For the vectors $\mathbf{a} = < 3, -4 >$, $\mathbf{b} = < 1, 3 >$, $\mathbf{c} = < 2, 1 >$, find:

(a) $|-4\mathbf{a} + 3\mathbf{b}|$

$-4\vec{a}+3\vec{b} = -4<3,-4>+3<1,3> = <-12+3, 16+9> = <-9,25>$

$|-4\vec{a}+3\vec{b}| = \sqrt{(-9)^2+25^2} = \sqrt{81+625} = \boxed{\sqrt{706}}$

(b) a unit vector in the direction opposite to $\mathbf{c}$

$-\vec{u} = -\frac{\vec{c}}{|\vec{c}|} = -\frac{<2,1>}{\sqrt{2^2+1^2}} = -\frac{1}{\sqrt{5}}<2,1> = \boxed{<-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}>}$

(c) a vector of length 3 in the direction of $\mathbf{b}$

a unit vector in the direction of $\vec{b}$ is $\frac{\vec{b}}{|\vec{b}|}$

$3\frac{\vec{b}}{|\vec{b}|} = 3\frac{<1,3>}{\sqrt{1+3^2}} = \frac{3}{\sqrt{10}}<1,3> = \boxed{<\frac{3}{\sqrt{10}}, \frac{9}{\sqrt{10}}>}$

(d) constants s and t such that $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$

$$\langle 2,1\rangle = s\langle 3,-4\rangle + t\langle 1,3\rangle$$

$$\langle 2,1\rangle = \langle 3s+t, -4s+3t\rangle$$

$$\begin{cases} 2 = 3s+t \longrightarrow t = 2-3s \\ 1 = -4s+3t \end{cases}$$

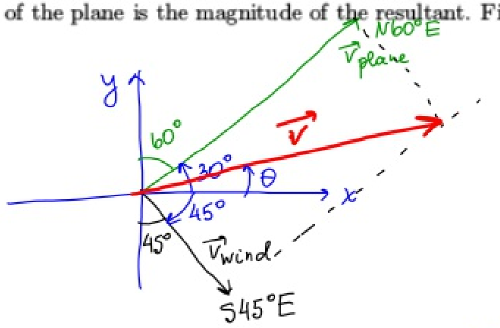
$$1 = -4s+3(2-3s)$$

$$1 = -4s+6-9s$$

$$-5 = -13s \Rightarrow \boxed{s = \frac{5}{13}}, \quad t = 2 - 3\cdot\frac{5}{13}$$

$$= 2 - \frac{15}{13} = \frac{26-15}{13} = \boxed{\frac{11}{13} = t}$$

6. Suppose that a wind is blowing in the direction S45°E at a speed of 60 km/h. A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 100 km/h. The true course, or track, of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The ground speed of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.



$|\vec{v}_{wind}| = 60$

$|\vec{v}_{plane}| = 100$

$\vec{v}_{wind} = 60 < \cos 45°, -\sin 45° >$
$= 60 < \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} >$

$= < 30\sqrt{2}, -30\sqrt{2} >$

$\vec{v}_{plane} = 100 < \cos 30°, \sin 30° >$
$= 100 < \frac{\sqrt{3}}{2}, \frac{1}{2} >$
$= < 50\sqrt{3}, 50 >$

resultant velocity $\vec{v} = \vec{v}_{wind} + \vec{v}_{plane}$
$= < 30\sqrt{2}, -30\sqrt{2} > + < 50\sqrt{3}, 50 >$
$= < 30\sqrt{2} + 50\sqrt{3}, -30\sqrt{2} + 50 > \approx < 129, 7.6 >$

ground speed $|\vec{v}| = \sqrt{129^2 + 7.6^2} \approx \boxed{129.2\ km/h}$

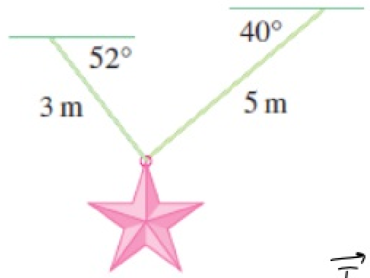
$|\vec{v}| = \sqrt{(30\sqrt{2}+50\sqrt{3})^2 + (-30\sqrt{2}+50)^2}$

$\tan\theta = \frac{7.6}{129}$, $\theta = \arctan\left(\frac{7.6}{129}\right) \approx 3.4°$

true course $N(90° - 3.4°)E$

$\boxed{N 86.6° E}$

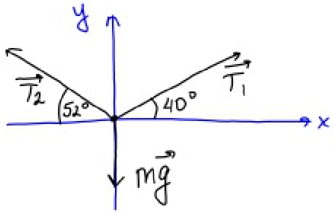
7. Ropes 3 m and 5 m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes, fastened at different heights, make angles of 52° and 40° with the horizontal. Find the magnitude of the tension in each wire.



$m = 5$

$g = 9.8$

$|\vec{T}_1| = ?, \quad |\vec{T}_2| = ?$



$\vec{T}_1 + \vec{T}_2 + m\vec{g} = \vec{0}$

$\vec{T}_1 = |\vec{T}_1| \langle \cos 40°, \sin 40° \rangle$

$\vec{T}_2 = |\vec{T}_2| \langle -\cos 52°, \sin 52° \rangle$

$m\vec{g} = \langle 0, -5(9.8) \rangle$

$|\vec{T}_1| \langle \cos 40°, \sin 40° \rangle + |\vec{T}_2| \langle -\cos 52°, \sin 52° \rangle + \langle 0, -5(9.8) \rangle = \langle 0, 0 \rangle$

Equate the corresponding components:

$$\begin{cases} |\vec{T}_1| \cos 40° - |\vec{T}_2| \cos 52° = 0 \longrightarrow |\vec{T}_2| = \dfrac{|\vec{T}_1| \cos 40°}{\cos 52°} \\ |\vec{T}_1| \sin 40° + |\vec{T}_2| \sin 52° - 5(9.8) = 0 \end{cases}$$

$$|\vec{T}_1| \sin 40° + \frac{|\vec{T}_1| \cos 40°}{\cos 52°} \sin 52° = 49$$

$$|\vec{T}_1| (\sin 40° + \cos 40° \tan 52°) = 49$$

$$|\vec{T}_1| = \frac{49}{\sin 40° + \cos 40° \tan 52°} \approx \boxed{30.2 = |\vec{T}_1|}$$

$$|\vec{T}_2| = \frac{30.2 \cos 40°}{\cos 52°} \approx \boxed{37.7 = |\vec{T}_2|}$$