Math 151/171 **WEEK in REVIEW 2** Fall 2016

1. Find $\mathbf{a} \cdot \mathbf{b}$

(a) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $150°$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta = 2(5)\cos 150° = 10\cos(180-30) = -10\cos 30°$
$= \boxed{-5\sqrt{3}}$

(b) $\mathbf{a} = -3\vec{i} + \vec{j}$, $\mathbf{b} = 2\vec{i} + 4\vec{j}$

$\langle -3, 1\rangle$ $\langle 2, 4\rangle$

$\vec{a} \cdot \vec{b} = (-3)(2) + 1(4) = -6 + 4 = \boxed{-2}$

2. Given the vectors $\mathbf{a} = \langle 1, -3\rangle$ and $\mathbf{b} = \langle -3, 4\rangle$. Find

(a) The scalar and vector projections of $\mathbf{a}$ onto $\mathbf{b}$

$$\text{comp}_{\vec{b}}\,\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{1(-3) + (-3)4}{\sqrt{(-3)^2 + 4^2}} = -\frac{15}{5} = \boxed{-3}$$

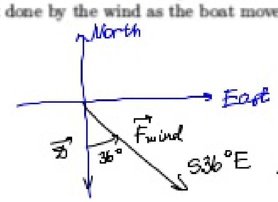
$$\text{proj}_{\vec{b}}\,\vec{a} = -\frac{15}{5^2}\,\vec{b} = -\frac{15}{25}\langle -3, 4\rangle = -\frac{3}{5}\langle -3, 4\rangle = \boxed{\left\langle \frac{9}{5}, -\frac{12}{5}\right\rangle}$$

(b) The scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$

$$\text{comp}_{\vec{a}}\,\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-15}{\sqrt{1^2 + (-3)^2}} = \boxed{\frac{-15}{\sqrt{10}}}$$

$$\text{proj}_{\vec{a}}\,\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\,\vec{a} = -\frac{15}{10}\langle 1, -3\rangle = \boxed{\left\langle -\frac{3}{2}, \frac{9}{2}\right\rangle}$$

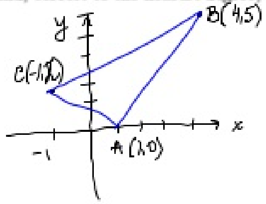
3. A boat sails south with the help of a wind blowing in the direction S36°E with magnitude 400 lb. Find the work done by the wind as the boat moves 110 ft. (Round your answer to the nearest whole number.)



$|\vec{F}| = 400$
$|\vec{D}| = 110$

$W = \vec{F} \cdot \vec{D} \cos 36°$
$= 400(110)(0.8)$
$\approx \boxed{35{,}597 \text{ ft-lb}}$

4. Find, correct to the nearest degree, the angle B of the triangle with the vertices $A(1,0)$, $B(4,5)$, $C(-1,2)$



$\angle ABC$ is an angle between vectors $\vec{AB}$ and $\vec{CB}$.

$\vec{AB} = \langle 3,5 \rangle$, $\vec{CB} = \langle 5,3 \rangle$

$$\cos\angle ABC = \frac{\vec{AB}\cdot\vec{CB}}{|\vec{AB}|\,|\vec{CB}|} = \frac{\langle 3,5\rangle\cdot\langle 5,3\rangle}{\sqrt{3^2+5^2}\cdot\sqrt{5^2+3^2}} = \frac{30}{34} = \frac{15}{17}$$

$$\angle ABC = \cos^{-1}\left(\frac{15}{17}\right) \approx \boxed{28^\circ}$$

5. Find a unit vector orthogonal to the vector $< -2, 4 >$.

$\vec{a} = \langle -2,4 \rangle$, $\vec{a}^{\perp} = \langle 4,2 \rangle$

$$\vec{u} = \frac{\vec{a}^{\perp}}{|\vec{a}^{\perp}|} = \frac{\langle 4,2\rangle}{\sqrt{4^2+2^2}} = \frac{\langle 4,2\rangle}{\sqrt{20}} = \left\langle \frac{4}{\sqrt{20}}, \frac{2}{\sqrt{20}} \right\rangle$$

$$= \boxed{\left\langle \frac{2}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle}$$

6. Find the value(s) of x such that the vectors $\overbrace{x\vec{i} + 3x\vec{j}}^{\vec{a}}$ and $\overbrace{x\vec{i} - 4\vec{j}}^{\vec{b}}$ are orthogonal.

$$\vec{a}\cdot\vec{b} = 0$$
$$x^2 + 3x(-4) = 0$$
$$x^2 - 12x = 0$$
$$x(x-12) = 0$$
$$x_1 = \boxed{0},\quad x_2 = \boxed{12}$$

7. Find the distance from the point (1,3) to the line $2x - 3y - 5 = 0$.

distance from the point (x_1, y_1) to the line $Ax + By + C = 0$ is

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$$

$$d = \left| \frac{2(1) - 3(3) - 5}{\sqrt{2^2 + (-3)^2}} \right| = \boxed{\frac{12}{\sqrt{13}}}$$

8. Find the distance between the parallel lines $y = 2x + 3$ and $y - 2x = 9$.

pick a point on one of two lines: $y=2x+3$ point $(0,3)$

Find the distance from $(0,3)$ to the line $y-2x=9$

or $-2x+y-9=0$

or $2x-y+9=0$.

$$dist = \left|\frac{2(0)-3+9}{\sqrt{2^2+(-1)^2}}\right| = \boxed{\frac{6}{\sqrt{5}}}$$

9. Find a Cartesian equation for the following parametric curves. Sketch the curve.

(a) $x = 1 - t^2$, $y = 1 - t$, $-1 \le t \le 1$

$t=1-y$, $x=1-(1-y)^2$

$x=1-(1-2y+y^2)$

$x = 2y-y^2$ parabola

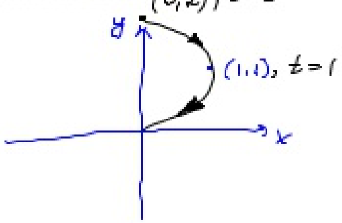
vertex @ $(1,1)$

$t=-1 \Rightarrow x=1-1=0$

$y=1-(-1)=2$

$t=0 \Rightarrow x=1$

$y=1$



(b) $x = 1 + \sin t$, $y = 2 + \cos t$, $\sin^2 x + \cos^2 y = 1$

$\sin t = x-1$, $\cos t = y-2$

$\underbrace{(x-1)^2}_{\sin^2 t}+\underbrace{(y-2)^2}_{\cos^2 t}=1$

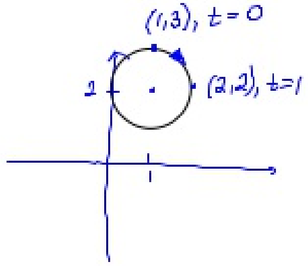
circle of radius 1 centered at $(1,2)$

$t=0 \Rightarrow x= 1+\sin 0 = 1$

$y=2+\cos 0 = 3$

$t=\frac{\pi}{2} \Rightarrow x=1+\sin\frac{\pi}{2}=2$

$y=2+\cos\frac{\pi}{2}=2$



(c) $x = \tan t$, $y = \cot^2 t$, $\frac{\pi}{6} \le t \le \frac{\pi}{3}$

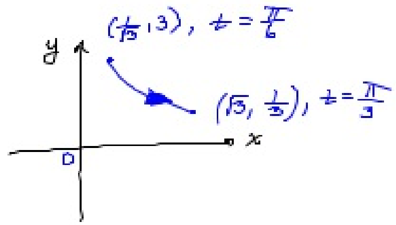
$y = \cot^2 t = \frac{1}{\tan^2 t} \Rightarrow y = \frac{1}{x^2}$

$t = \frac{\pi}{6}$: $x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$y = \cot^2\frac{\pi}{6} = 3$

$t = \frac{\pi}{3}$: $x = \tan\frac{\pi}{3} = \sqrt{3}$

$y = \cot^2\frac{\pi}{3} = \frac{1}{3}$



10. An object is moving in the xy-plane and its position after t seconds is $\mathbf{r}(t) = \langle t^2 + t, t - 4 \rangle$.

(a) At what time is the object at the point (12,-1).

Find t such that $t^2 + t = 12$ and $t - 4 = -1$

$\boxed{t = 3}$

Plug into the first equation: $3^2 + 3 = 9 + 3 = 12$.

(b) Does the object pass through the point (4,8)?

it does if there is t such that

$\begin{cases} t^2 + t = 4 \\ t - 4 = 8 \end{cases} \Rightarrow t = 12$, plug into the 1st eqn:

$12^2 + 12 = 144 + 12 = 156 \neq 4$

$\boxed{NO}$

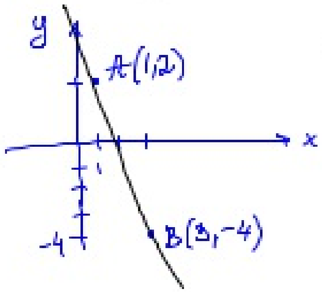
(c) Find an equation in x and y whose graph is the path of the object.

$x = t^2 + t$, $y = t - 4$, eliminate t

$t = y + 4$

$\boxed{x = (y+4)^2 + y + 4}$

11. Find a vector equation of the line containing the points (1,2) and (3,-4).



line is parallel to the vector $\vec{AB} = \langle 3-1, -4-2 \rangle$

$\Rightarrow \langle 2, -6 \rangle$

either $\boxed{\vec{r}(t) = \langle 1,2 \rangle + t\langle 2,-6 \rangle}$ or $\boxed{\vec{r}(t) = \langle 3,-4 \rangle + t\langle 2,-6 \rangle}$

$\vec{v} = \langle 1,-5 \rangle$

12. Find parametric equations of the line passing through the point (-1,1) and parallel to the vector $\vec{i} - 5\vec{j}$.

vector eqn. $\vec{r}(t) = \langle -1, 1 \rangle + t\langle 1,-5 \rangle$

$\langle x, y \rangle = \langle -1, 1 \rangle + t\langle 1,-5 \rangle$

$\boxed{\begin{aligned} x &= -1+t \\ y &= 1-5t \end{aligned}}$ parametric equations

13. Determine whether the lines $\mathbf{r}(t) = (-4+2t)\vec{i} + (5+t)\vec{j}$ and $\mathbf{r}(t) = (2+3t)\vec{i} + (4-6t)\vec{j}$ are parallel, perpendicular or neither. If they are not parallel, find their point of intersection.

L_1: $\vec{r}(t) = \langle -4+2t, 5+t \rangle$, L_2: $\langle 2+3t, 4-6t \rangle$

L_1 is $\parallel$ to $\langle 2, 1 \rangle$ L_2 is $\parallel$ to $\langle 3, -6 \rangle$

$\langle 2, 1 \rangle \cdot \langle 3, -6 \rangle = 6 - 6 = 0$ $\boxed{\text{perpendicular}}$

Point of intersection: line L_1: $\begin{cases} x = -4+2t \\ y = 5+t \end{cases}$ line L_2: $\begin{cases} x = 2+3s \\ y = 4-6s \end{cases}$

$$\begin{cases} -4+2t = 2+3s \\ 5+t = 4-6s \end{cases}$$

$$\begin{cases} 2t = 3s+6 \\ t = -6s-1 \end{cases}$$

$-12s - 2 = 3s + 6$

$-15s = 8 \Rightarrow s = -\frac{8}{15}$.

Point $x = 2 + 3\left(-\frac{8}{15}\right) = \frac{30-24}{15} = \frac{6}{15} = \frac{2}{5}$

$y = 4 - 6\left(-\frac{8}{15}\right) = 4 + \frac{48}{15} = \frac{108}{15} = \frac{36}{5}$

$$\boxed{\left(\frac{2}{5}, \frac{36}{5}\right)}$$

14. Find all holes and vertical asymptote(s) for the graph of

$$g(x) = \frac{(x^2+5x)(x-2)}{(x+1)(x^2+4x-5)}$$

and determine the behavior of the function near the vertical asymptotes.

$$g(x) = \frac{(x^2+5x)(x-2)}{(x+1)(x+5)(x-1)} = \frac{x\cancel{(x+5)}(x-2)}{(x+1)\cancel{(x+5)}(x-1)} = \frac{x(x-2)}{(x+1)(x-1)}$$

hole @ $x=-5$

vertical asymptotes: $x=1$ and $x=-1$

$$\lim_{x\to 1^-} g(x) = \frac{(+)(-)}{(+)(-)} = (+) = \infty$$

(say 0.99)

$$\lim_{x\to 1^+} g(x) = \frac{(+)(-)}{(+)(+)} = (-) = -\infty$$

(say 1.01)

$$\lim_{x\to -1^-} g(x) = \frac{(-)(-)}{(-)(-)} = (+) = \infty$$

(say -1.01)

$$\lim_{x\to -1^+} g(x) = \frac{(-)(-)}{(+)(-)} = (-) = -\infty$$

(say -0.99)

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15. For the function g whose graph is given, state the value of the given quantity, if it exists.

(a) $\lim_{x\to -2^-} g(x)$ = -1

(b) $\lim_{x\to -2^+} g(x)$ = 2

(c) $\lim_{x\to -2} g(x)$ DNE

(d) $g(-2)$ = 2

(e) $\lim_{x\to 0} g(x)$ = 0

(f) $g(0)$ = 0

(g) $\lim_{x\to 2^-} g(x)$ = 2

(h) $\lim_{x\to 2^+} g(x)$ = 6

(i) $g(2)$ = 6

(j) $\lim_{x\to 4^-} g(x)$ = 6

(k) $\lim_{x\to 4^+} g(x)$ DNE (g(x) oscillates between 4 and 8, does not approach to any point

(l) $\lim_{x\to 4} g(x)$ DNE