1. Calculate the following limits or state why the limit does not exist. Do not use the L'Hospital's Rule.

   (a) $\lim_{x\to 4} \sqrt{x+\sqrt{x}}$

   (b) $\lim_{x\to 5} \dfrac{5x-x^2}{x^2-4x-5}$

   (c) $\lim_{h\to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h}$

   (d) $\lim_{x\to 3} \dfrac{x-\sqrt{4x-3}}{x^2-9}$

   (e) $\lim_{x\to 0} \left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

(f) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{|x - 2|}$

(g) $\lim_{t \to 5} \left\langle \dfrac{2t - 10}{t - 5}, \dfrac{5 - t}{t^2 - 4t - 5} \right\rangle$

(h) $\lim_{x \to 0} x^4 \cos \dfrac{1}{x^2}$

2. If $2x - 2 \leq f(x) \leq x^2 - 2x + 2$ for $x \geq 0$, find $\lim_{x \to 2} f(x)$.

3. Let

$$f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0 \\ 3 - x, & \text{if } 0 \leq x < 3 \\ (x-3)^2, & \text{if } x > 3 \end{cases}$$

Evaluate each limit if exists.

(a) $\lim_{x \to 0} f(x)$

(b) $\lim_{x \to 3} f(x)$

4. Find the x-value at which f is discontinuous and determine whether f is continuous from the right, or from the left, or neither.

$$f(x) = \begin{cases} 1 + x^2, & \text{if } x \leq 0 \\ 4 - x, & \text{if } 0 < x \leq 4 \\ (x-4)^2, & \text{if } x > 4 \end{cases}$$

5. Find the value(s) of x where the function $f(x)$ is discontinuous. If the discontinuity, $x = a$, is removable, find a function g that agrees with f for all values of x and is continuous at $x = a$.

(a) $f(x) = \dfrac{x - 4}{x^2 + x - 20}$

(b) $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 6}$

6. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & \text{if } x < 2 \\ ax^2 - bx + 3, & \text{if } 2 \le x < 3 \\ 4x - a + b, & \text{if } x \ge 3 \end{cases}$$

7. Use the Intermediate Value Theorem to show that there is a root of the equation $x^4 + x = 3$ in the interval $(1, 2)$.

8. Find the limit.

(a) $\lim\limits_{x \to \infty} \dfrac{x^2 - 5x + 1}{3x + 7}$

(b) $\lim_{x\to\infty} \dfrac{x^2+x-4}{x^3-2x+1}$

(c) $\lim_{x\to-\infty} \dfrac{2x^3+3x^2-3x+7}{x^3-16x+5}$

(d) $\lim_{x\to\infty} \left(\sqrt{x^2+x-1}-\sqrt{x^2-x}\right)$

(e) $\lim_{x\to-\infty} \left(x+\sqrt{x^2+2x}\right)$

9. Find the vertical and horizontal asymptotes (if any) for the function $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 6}$.