1. Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$. $= \sin\left(\pi + \frac{5\pi}{12}\right) = -\sin\frac{5\pi}{12}$

$\frac{5\pi}{12} = \frac{5\pi}{12}\left(\frac{180}{\pi}\right)^{\circ} = \frac{150^{\circ}}{2} = 75^{\circ} = 45^{\circ} + 30^{\circ}$

$-\sin 75^{\circ} = -\sin(45^{\circ}+30^{\circ}) = -[\sin 45^{\circ}\cos 30^{\circ} + \cos 45^{\circ}\sin 30^{\circ}]$

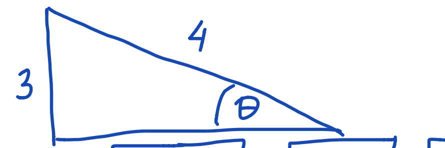
$= -\left(\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}\right) = -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}+1}{2}\right) = \boxed{-\frac{\sqrt{2}(\sqrt{3}+1)}{4}}$

2. If $\csc\theta = -\frac{4}{3}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, find $\cos\theta$, $\sin\theta$, $\tan\theta$, $\cot\theta$.

IV quadrant, only $\cos\theta > 0$

$\csc\theta = -\frac{4}{3} \Rightarrow \frac{1}{\sin\theta} = -\frac{4}{3} \rightarrow \boxed{\sin\theta = -\frac{3}{4}}$

$\boxed{\cos\theta = \frac{\sqrt{7}}{4}}$ $\boxed{\tan\theta = \frac{-3}{\sqrt{7}}}$ $\boxed{\cot\theta = -\frac{\sqrt{7}}{3}}$



$\sqrt{4^2-3^2} = \sqrt{16-9} = \sqrt{7}$

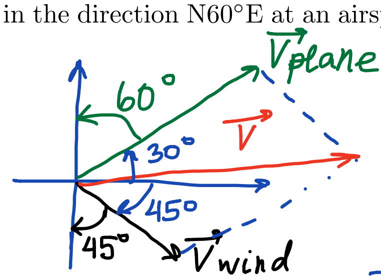
3. A constant force $\mathbf{F} = 5\vec{\imath} + 6\vec{\jmath}$ moves an object along a straight line from the point A(-1,2) to the point B(2,3). Find the work done by the force $\mathbf{F}$.

$W = \vec{F}\cdot\vec{AB}$

$\vec{AB} = \langle 2-(-1), 3-2\rangle = \langle 3,1\rangle$

$W = \langle 5,6\rangle\cdot\langle 3,1\rangle = 5(3)+6(1) = 15+6 = \boxed{21}$

4. Suppose that a wind is blowing in the direction S45°E at a speed of 60 km/h. A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 100 km/h. Find the ground speed of the plane.



$|\vec{V}_{wind}| = 60$

$|\vec{V}_{plane}| = 100$

$\vec{V}_{wind} = 60\langle\cos 45^{\circ}, -\sin 45^{\circ}\rangle$
$= 60\left\langle\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle = \langle 30\sqrt{2}, -30\sqrt{2}\rangle$

$\vec{V}_{plane} = 100\langle\cos 30^{\circ}, \sin 30^{\circ}\rangle$
$= 100\left\langle\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle = \langle 50\sqrt{3}, 50\rangle$

$\vec{V} = \vec{V}_{wind} + \vec{V}_{plane} = \langle 30\sqrt{2}, -30\sqrt{2}\rangle + \langle 50\sqrt{3}, 50\rangle$

$= \langle 30\sqrt{2}+50\sqrt{3}, -30\sqrt{2}+50\rangle$

Ground speed $|\vec{V}| = \boxed{\sqrt{(30\sqrt{2}+50\sqrt{3})^2 + (50-30\sqrt{2})^2}}$

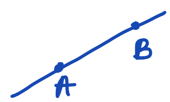
5. Find the scalar and vector projections of the vector $2\vec{\imath} - 3\vec{\jmath}$ onto the vector $\vec{\imath} + 6\vec{\jmath}$.

$\vec{a} = \langle 2,-3\rangle$, $\vec{b} = \langle 1,6\rangle$

$\text{comp}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{\langle 2,-3\rangle\cdot\langle 1,6\rangle}{\sqrt{1^2+6^2}} = \frac{2-3(6)}{\sqrt{37}} = \boxed{\frac{-16}{\sqrt{37}}}$

$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b} = -\frac{16}{37}\langle 1,6\rangle = \boxed{\left\langle -\frac{16}{37}, -\frac{96}{37}\right\rangle}$

6. Find the vector, parametric, and the Cartesian equations for the line passing through the points $A(1,-3)$ and $B(2,1)$.



the line is parallel to the vector $\vec{AB}$

$\vec{AB} = \langle 2-1, 1-(-3)\rangle = \langle 1, 4\rangle$

vector equations: $\vec{r}(t) = \langle 1,-3\rangle + t\langle 1,4\rangle$ or $\vec{r}(t) = \langle 2,1\rangle + t\langle 1,4\rangle$

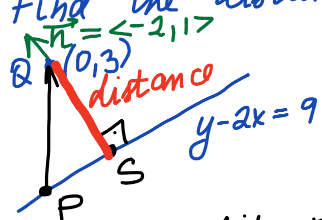
parametric equations: $x(t) = 1+t$, $y(t) = -3+4t$ or $x(t) = 2+t$, $y(t) = 1+4t$

Cartesian equations: $t = x-1$, $y = -3+4(x-1)$, $y = -3+4x-4$, $\boxed{y = 4x-7}$ or $t = x-2$, $y = 1+4(x-2)$, $y = 1+4x-8$, $\boxed{y = 4x-7}$

7. Find the distance between the parallel lines $y = 2x+3$ and $y-2x = 9$.

Pick a point on one of the lines: $(0,3)$ on $y = 2x+3$

Find the distance from $Q(0,3)$ to $y-2x=9$



$$\text{distance} = \left|\text{comp}_{\vec{n}} \vec{QP}\right| = \left|\frac{3-2(0)-9}{\sqrt{1+(-2)^2}}\right| = \left|-\frac{6}{\sqrt{5}}\right| = \boxed{\frac{6}{\sqrt{5}}}$$

P is an arbitrary point on the line $y-2x=9$

8. Given the parametric curve $x(t) = 1+\cos t$, $y(t) = 1-\sin^2 t$.

(a) Find a Cartesian equation for this curve.

$\cos t = x-1$, $\sin^2 t = 1-y$

use trig. identity $\underbrace{\sin^2 t}_{1-y} = 1 - \underbrace{\cos^2 t}_{(x-1)^2}$

$1-y = 1-(x-1)^2$ or $\boxed{y = (x-1)^2}$ parabola

Domain: $-1 \le \cos t \le 1$

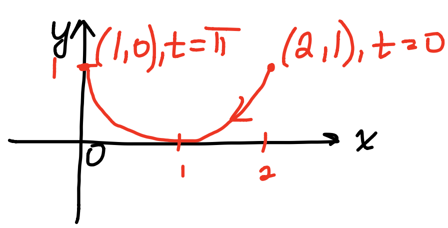
$0 \le 1+\cos t \le 2$

$\boxed{0 \le x \le 2}$

(b) Does the parametric curve go through the point (1,0)? If yes, give the value(s) of t.

Find t such that $\begin{cases} 1+\cos t = 1 \\ 1-\sin^2 t = 0 \end{cases} \Rightarrow \cos t = 0$, $\boxed{t = \frac{\pi}{2} + \pi n}$

(c) Sketch the graph of the parametric curve on the interval $0 \le t \le \pi$, include the direction of the path.



$0 \le t \le \pi$

$t=0$: $x(0) = 1+\cos 0 = 2$, $y(0) = 1-\sin^2 0 = 1$ $\Big\}(2,1)$

$t=\pi$: $x(\pi) = 1+\cos\pi = 0$, $y(\pi) = 1-\sin^2\pi = 1$ $\Big\}(0,1)$

9. Evaluate the limit (do no use the L'Hospital's Rule):

(a) $\lim_{x \to 5} \frac{x^2 - 5x + 10}{x^2 - 25} = \frac{5^2 - 5(5) + 10}{5^2 - 25} = \frac{10}{0} = \boxed{DNE}$

$\lim_{x \to 5^-} \frac{x^2 - 5x + 10}{x^2 - 25} = \frac{-}{-} = \infty$

$\lim_{x \to 5^+} \frac{x^2 5x - 10}{x^2 - 25} = \frac{-}{+} = \infty$

(b) $\lim_{x \to 7} \frac{(2 - \sqrt{x-3})(2+\sqrt{x-3})}{(x^2 - 49)(2+\sqrt{x-3})} = \lim_{x \to 7} \frac{4 - (x-3)}{(x^2 - 49)(2+\sqrt{x-3})} = \lim_{x \to 7} \frac{7 - x}{(x-7)(x+7)(2+\sqrt{x-3})}$

$= \lim_{x \to 7} \frac{-\cancel{(x-7)}}{\cancel{(x-7)}(x+7)(2+\sqrt{x-3})} = \lim_{x \to 7} \frac{-1}{(x+7)(2+\sqrt{x-3})} =$

$= -\frac{1}{(7+7)(2+\sqrt{7-3})} = -\frac{1}{14(4)} = \boxed{-\frac{1}{56}}$

(c) $\lim_{t \to 1} \left\langle \frac{t^2 - 2t + 1}{t - 1}, \frac{\sqrt{t} - 1}{t^2 - 1} \right\rangle = < \lim_{t \to 1} \frac{t^2 - 2t + 1}{t - 1}, \lim_{t \to 1} \frac{(\sqrt{t} - 1)(\sqrt{t}+1)}{(t^2 - 1)(\sqrt{t}+1)} >$

$= < \lim_{t \to 1} \frac{(t-1)^{\cancel{2}}}{\cancel{t-1}}, \lim_{t \to 1} \frac{\cancel{t-1}}{\cancel{(t-1)}(t+1)(\sqrt{t}+1)} > = < \lim_{t \to 1} (t-1), \lim_{t \to 1} \frac{1}{(t+1)(\sqrt{t}+1)} >$

$= \boxed{< 0, \frac{1}{4} >}$

(d) $\lim_{x \to -2} \frac{x^2 - 4}{|x+2|}$ $\boxed{DNE}$

$|x+2| = \begin{cases} x+2, & \text{if } x \geq -2 \\ -(x+2), & \text{if } x < -2 \end{cases}$

$\lim_{x \to -2^-} \frac{x^2 - 4}{|x+2|} = \lim_{x \to -2} \frac{x^2 - 4}{-(x+2)} = \lim_{x \to -2} \frac{(x-2)\cancel{(x+2)}}{-\cancel{(x+2)}}$

$= \lim_{x \to -2} \frac{(x-2)}{-1} = 4$

$\lim_{x \to -2^+} \frac{x^2 - 4}{|x+2|} = \lim_{x \to -2} \frac{x^2 - 4}{x+2} = \lim_{x \to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$= \lim_{x \to -2} (x - 2) = -4$

$-4 \neq 4$

(e) $\lim_{x\to 0}\left(\frac{1}{x\sqrt{x+1}}-\frac{1}{x}\right) = \lim_{x\to 0} \frac{(1-\sqrt{x+1})(1+\sqrt{x+1})}{(x\sqrt{x+1})(1+\sqrt{x+1})} = \lim_{x\to 0} \frac{1-(x+1)}{x\sqrt{x+1}(1+\sqrt{x+1})}$

$= \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}\sqrt{x+1}(1+\sqrt{x+1})} = \frac{1}{1(1+1)} = \boxed{\frac{1}{2}}$

(f) $\lim_{y\to\infty} \frac{7y^3+4y}{2y^3-y^2+3} = \lim_{y\to\infty} \frac{\cancel{y^3}(7+\frac{4}{y^2}\to 0)}{\cancel{y^3}(2-\frac{1}{y}\to 0+\frac{3}{y^2}\to 0)} = \boxed{\frac{7}{2}}$

(g) $\lim_{x\to -\infty} (x+\sqrt{x^2+2x})\frac{(x-\sqrt{x^2+2x})}{x-\sqrt{x^2+2x}} = \lim_{x\to -\infty} \frac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}}$

$= \lim_{x\to -\infty} \frac{-2x}{x-\sqrt{x^2(1+\frac{2}{x}\to 0)}} = \lim_{x\to -\infty} \frac{-2x}{x-\sqrt{x^2}} = \lim_{x\to -\infty} \frac{-2x}{x+x} = \boxed{-1}$

$\sqrt{x^2} = -x$ if $x<0$

10. (a) Find and classify all points of discontinuity for the function

$$f(x) = \begin{cases} x^2+1 & , \text{ if } x < 2, \\ x+2 & , \text{ if } x \geq 2. \end{cases}$$

x^2+1 continuous for all x

$x+2$ continuous for all x

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2}(x^2+1) = 5$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2}(x+2) = 4$

$4 \neq 5 \to$ jump discontinuity @ $x=2$

(b) Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2+4}{3x^2-3}$. $= \frac{x^2+4}{3(x^2-1)} = \frac{x^2+4}{3(x-1)(x+1)}$

V.A. $\boxed{x=1,\ x=-1}$

$$\lim_{x\to\infty} \frac{x^2+4}{3x^2-3} = \lim_{x\to\infty} \frac{\cancel{x^2}(1+\cancel{\frac{4}{x^2}})}{\cancel{x^2}(3-\cancel{\frac{3}{x^2}})} = \frac{1}{3} \quad \boxed{H.A.\ y=\frac{1}{3}}$$

11. Use the Intermediate Value Theorem to show that there is a root of the equation $x^3 - 3x + 1 = 0$ in the interval (1,2).

$f(x) = x^3-3x+1$ – continuous on $(-\infty, \infty)$

$f(1) = 1-3+1 = -1 < 0$

$f(2) = 8-6+1 = 3 > 0$

since $f(x)$ is continuous on $(1,2)$ and $f(1)<0$, $f(2)>0$, then, by the Intermediate Value Thm, there is a number $1<c<2$ such that $f(c)=0$.

12. Find $f'(x)$ by using the definition of derivative if

(a) $f(x) = (3-x)^2$, $f(x+h) = (3-(x+h))^2 = (3-x-h)^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(3-x-h)^2-(3-x)^2}{h}$$

$$= \lim_{h\to 0} \frac{(3-x-h+(3-x))(3-x-h-(3-x))}{h} = \lim_{h\to 0} \frac{(6-2x-h)(-\cancel{h})}{\cancel{h}} = -(6-2x) = \boxed{2x-6}$$

(b) $f(x) = \sqrt{x-2}$, $f(x+h) = \sqrt{x+h-2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h-2}-\sqrt{x-2})(\sqrt{x+h-2}+\sqrt{x-2})}{h(\sqrt{x+h-2}+\sqrt{x-2})}$$

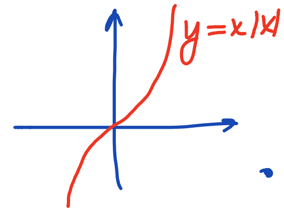
$$= \lim_{h\to 0} \frac{x+h-2-(x-2)}{h(\sqrt{x+h-2}+\sqrt{x-2})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2}+\sqrt{x-2})} = \boxed{\frac{1}{2\sqrt{x-2}}}$$

(c) $f(x) = \frac{1}{x+1}$, $f(x+h) = \frac{1}{x+h+1}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h+1} - \frac{1}{x+1}\right) = \lim_{h\to 0} \frac{x+1-(x+h+1)}{h(x+1)(x+h+1)}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x+1)(x+h+1)} = \boxed{-\frac{1}{(x+1)^2}}$$

13. Let $f(x) = x|x|$ $= \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$



• no corners
• no points of discontinuity
• no vertical tangents

(a) For what values of x is f differentiable?

differentiable for all x.

(b) Find a formula for f'.

$$f'(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases} = 2|x|$$

slope is 3

14. At what point on the curve $y = x^{3/2}$ is the tangent line parallel to the line $3x - y + 6 = 0$.

slope of the tangent line should be 3.

$$\text{slope} = f'(x) = (x^{3/2})' = \frac{3}{2}x^{1/2} = 3$$

$$x^{1/2} = 2$$

$$x = 4$$

corresponding $y = 4^{3/2} = 8$

$\boxed{(4, 8)}$

15. Find the tangent vector and parametric equations for the line tangent to the curve $\vec{r}(t) =< t^2 + 2t, t^3 - t >$ at the point corresponding to $t = 1$.

tangent vector $\vec{v}(t) = \lim_{t \to 1} \dfrac{\vec{r}(t) - \vec{r}(1)}{t-1} = \lim_{t \to 1} \dfrac{< t^2+2t, t^3-t > - < 1+2, 1-1 >}{t-1}$

$$= \lim_{t \to 1} < \frac{t^2+2t-3}{t-1}, \frac{t^3-t}{t-1} > = < \lim_{t \to 1} \frac{\cancel{(t-1)}(t+3)}{\cancel{t-1}}, \lim_{t \to 1} \frac{t(t^2-1)}{t-1} >$$

$$= < \lim_{t \to 1} (t+3), \lim_{t \to 1} \frac{t\cancel{(t-1)}(t+1)}{\cancel{t-1}} > = < 4, \lim_{t \to 1} t(t+1) > = < 4, 2 >$$

Vector equation of the tangent line: $\vec{r}(t) = \vec{r}(1) + t\vec{v}(1) = \boxed{< 3, 0 > + t < 4, 2 >}$

parametric equations: $\boxed{\begin{array}{l} x(t) = 3 + 4t \\ y(t) = 2t \end{array}}$

16. The displacement of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$ (t is in seconds). Find the velocity of the object when $t = 1$.

$$\vec{v}(t) = \lim_{t \to 1} \frac{s(t) - s(1)}{t-1} = \lim_{t \to 1} \frac{1 + 2t + \frac{t^2}{4} - \left(1 + 2 + \frac{1}{4}\right)}{t-1} = \lim_{t \to 1} \frac{1 + 2t + \frac{t^2}{4} - \frac{13}{4}}{t-1}$$

$$= \lim_{t \to 1} \frac{\frac{t^2}{4} + 2t - \frac{9}{4}}{t-1} = \lim_{t \to 1} \frac{t^2 + 8t - 9}{4(t-1)} = \lim_{t \to 1} \frac{\cancel{(t-1)}(t+9)}{4\cancel{(t-1)}} = \frac{10}{4} = \boxed{\frac{5}{2}}$$

17. The vector function $\vec{r}(t) = (t^2 - 4t)\vec{\imath} + (2t+1)\vec{\jmath}$ represents the position of a particle at time t.

(a) Find the velocity of the particle when $t = 1$

$$\vec{V}(1) = \lim_{t\to 1} \frac{\vec{V}(t) - \vec{V}(1)}{t-1} = \lim_{t\to 1} \frac{\langle t^2-4t, 2t+1\rangle - \langle 1-4, 2+1\rangle}{t-1}$$

$$= \lim_{t\to 1} \frac{\langle t^2-4t+3, 2t-2\rangle}{t-1} = \left\langle \lim_{t\to 1} \frac{t^2-4t+3}{t-1}, \lim_{t\to 1} \frac{2t-2}{t-1} \right\rangle$$

$$= \left\langle \lim_{t\to 1} \frac{\cancel{(t-1)}(t-3)}{\cancel{t-1}}, \lim_{t\to 1} \frac{2\cancel{(t-1)}}{\cancel{t-1}} \right\rangle = \boxed{\langle -2, 2\rangle}$$

(b) Find the speed of the particle when $t = 1$

$$\text{speed} = |\langle -2, 2\rangle| = \sqrt{2^2+2^2} = \sqrt{8} = \boxed{2\sqrt{2}}$$