1. Find the linear and quadratic approximation for the function $f(x) = \sin x$ at $a = \pi/6$.

2. Use differentials to approximate $(1.97)^6$.

3. Use differentials to estimate the amount of paint needed to apply a coat of paint .05 cm thick to a hemispherical dome with diameter 50 m.

4. Find the limit

(a) $\lim_{x \to -\infty} \pi^x$

(b) $\lim_{x \to \infty} \left(\frac{3}{\pi}\right)^x$

(c) $\lim_{x \to -\infty} \left(\frac{3}{\pi}\right)^x$

(d) $\lim_{x \to \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(e) $\lim_{x \to 3^-} \left(\frac{1}{7}\right)^{\frac{x}{3-x}}$

(f) $\displaystyle\lim_{x \to \frac{\pi}{2}^+} \frac{2}{1 + e^{\tan x}}$

(g) $\displaystyle\lim_{t \to 0} \frac{\sin^2 3t}{t^2 \cos 5t}$

(h) $\displaystyle\lim_{x \to 0} \frac{\tan 3x}{\tan 5x}$

(i) $\displaystyle\lim_{x \to -2} \frac{\tan \pi x}{x + 2}$

5. Find an equation of the tangent line to the curve $2e^{xy} = x + y$ at the point (0,2).

6. Find the values of a and b that make the function

$$f(x) = \begin{cases} ax^2 + x + 1, & \text{if } x \leq 1 \\ bx - 1, & \text{if } x > 1 \end{cases}$$

differentiable everywhere. Find $f'(x)$.

7. If $f(x) = \sin(g(x))$, find $f'(2)$ given that $g(2) = \dfrac{\pi}{3}$ and $g'(2) = \dfrac{\pi}{4}$.

8. At what point on the curve $f(x) = 36\sqrt{x}$ is the tangent line parallel to the line $9x - y + 2 = 0$?

9. An object is moving along a straight path. The position of the object at time t is given by $s(t) = 2t^3 - 9t^2 + 12t + 1$, where t is measured in seconds and $s(t)$ is measured in feet. Find the total distance traveled in the first 2 seconds.

10. Find the derivative

    (a) $f(x) = x^2 \cot(3x)$

    (b) $f(x) = \dfrac{e^{\sqrt{x}}}{x + \sqrt{x}}$

(c) $f(x) = \tan^3(e^{-x} + ex - x^e)$

(d) $f(x) = \left(\dfrac{x^3 + 3x}{x^2 - 4x + 1}\right)^{5/2}$

11. Find $f^{(58)}(x)$ if $f(x) = e^{-2x} + \cos(3x)$.

12. The vector function $\mathbf{r}(t) =< t + e^{4t}, -t\cos(2t) >$, $0 \leq t \leq 2\pi$, represents the position of a particle at time t. Find the velocity acceleration vectors of the object at $t = \dfrac{\pi}{4}$.

13. At what point(s) does the curve parametrized by $x = t^2 - 6t + 5$, $y = t^2 + 4t + 3$ have a horizontal or vertical tangent?

14. Water is leaking out of an inverted conical tank at a rate of 1 m^3/min. The tank has height 6 m and the diameter at the top is 4 m. At what rate is the water level changing when the height of the water is 3 m?

15. A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 15 min after woman starts walking.

16. A trapezoid has a base with length 10 inches. The length of the top of the trapezoid is decreasing at a rate of 1 in/min while the height of the trapezoid is increasing at a rate of 2 in/min. At what rate is the area of the trapezoid changing when the height is 3 inches and the area of the trapezoid is 24 in^2?