1. Find the exact value of the expression.

$y = \boxed{\frac{\pi}{3}}$

(a) $\arcsin \frac{\sqrt{3}}{2} = y = \boxed{\frac{\pi}{3}}$ means that $\sin y = \frac{\sqrt{3}}{2}$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\boxed{\arccos(-x) = \pi - \arccos x}$

$0 \le \cos^{-1} x \le \pi$

(b) $\arccos\left(-\frac{1}{2}\right) = \pi - \arccos\frac{1}{2} = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

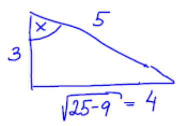
(c) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

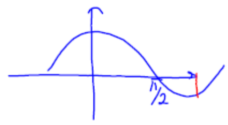
(d) $\tan^{-1}\sqrt{3} = \boxed{\pi/3}$

(e) $\sin\left(\underbrace{\cos^{-1}\left(-\frac{3}{5}\right)}_{x}\right) = \sin x$

$x = \cos^{-1}\left(\overbrace{-\frac{3}{5}}^{y}\right)$

$\cos x = -\frac{3}{5}$





$\frac{\pi}{2} \le x \le \pi$

$$\boxed{\begin{array}{c} -1 \le y \le 1 \\ 0 \le \cos^{-1}(y) \le \pi \end{array}}$$

$\frac{\pi}{2} \le x \le \pi$ (2nd quadrant, $\sin > 0$)

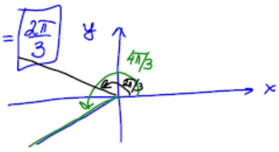
$\boxed{\sin x = \frac{4}{5}}$

Domain for $\sin^{-1}x$ is $[-1,1]$, 3 is not in the domain

(f) $\sin(\arcsin 3)$ DNE

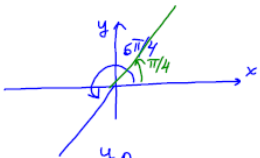
(g) $\cos^{-1}\left(\cos\dfrac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

$\cos\frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

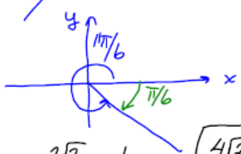


(h) $\tan^{-1}\left(\tan\dfrac{5\pi}{4}\right) = \tan^{-1}(1) = \boxed{\frac{\pi}{4}}$

$\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$



(i) $\sin^{-1}\left(\sin\dfrac{11\pi}{6}\right) = \boxed{-\frac{\pi}{6}}$

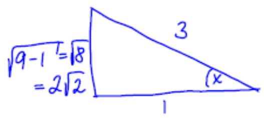


(j) $\sin\left(2\cos^{-1}\dfrac{1}{3}\right) = \sin 2x = 2\sin x\cos x = 2\cdot\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} = \boxed{\frac{4\sqrt{2}}{9}}$

$\cos^{-1}\frac{1}{3} = x$

$\cos x = \frac{1}{3}$

$\sin x = \frac{2\sqrt{2}}{3}$

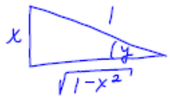


2. Simplify each expression.

(a) $\tan(\underbrace{\sin^{-1} x}_{y}) = \tan y = \boxed{\dfrac{x}{\sqrt{1-x^2}}}$

$y = \sin^{-1} x$

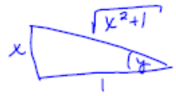
$x = \sin y$



(b) $\cos(\underbrace{\tan^{-1} x}_{y}) = \cos y = \boxed{\dfrac{1}{\sqrt{x^2+1}}}$

$y = \tan^{-1} x$

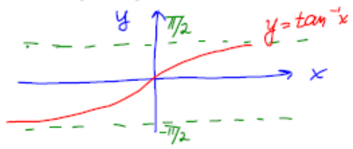
$x = \tan y$



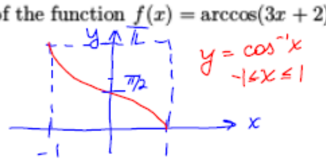
3. Find the limit.

(a) $\lim\limits_{x\to\infty} \sin^{-1}\left(\dfrac{x^2-1}{2x^2+4}\right) = \sin^{-1}\left(\lim\limits_{x\to\infty} \dfrac{x^2-1}{2x^2+4}\right) = \sin^{-1}\left(\lim\limits_{x\to\infty} \dfrac{\cancel{x^2}(1-\overset{\to 0}{1/x^2})}{\cancel{x^2}(2+\underset{\to 0}{4/x^2})}\right) = \sin^{-1}\left(\dfrac{1}{2}\right) = \boxed{\dfrac{\pi}{6}}$

(b) $\lim\limits_{x\to\infty} \tan^{-1}\left(\dfrac{x^2}{2-x}\right) = \tan^{-1}\left(\lim\limits_{x\to\infty} \dfrac{x^2}{2-x}\right) = \lim\limits_{y\to-\infty} \tan^{-1} y = \boxed{-\dfrac{\pi}{2}}$



4. Find the domain of the function $f(x) = \arccos(3x+2)$.



$-1 \le 3x+2 \le 1$

$-1-2 \le 3x \le 1-2$

$\dfrac{-3}{3} \le \dfrac{3x}{3} \le \dfrac{-1}{3}$

$\boxed{-1 \le x \le -\dfrac{1}{3}}$

Domain

$\boxed{(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}}$ $\boxed{(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}}$, $\boxed{(\arctan x)' = \frac{1}{1+x^2}}$, $\boxed{(\text{arccot}\, x)' = -\frac{1}{1+x^2}}$

5. Find the derivative.

(a) $y = \tan^{-1}(2x+1)$

$$y' = \frac{1}{1+(2x+1)^2}(2x+1)'$$
$$= \boxed{\frac{2}{1+(2x+1)^2}}$$

2

(b) $y = \sqrt{x}\sin^{-1}(x^3)$

$$y' = \frac{1}{2}x^{-1/2}\sin^{-1}(x^3) + x^{1/2}\frac{1}{\sqrt{1-(x^3)^2}}(x^3)'$$
$$= \frac{1}{2}x^{-1/2}\sin^{-1}(x^3) + \frac{x^{1/2}(3x^2)}{\sqrt{1-x^6}} = \boxed{\frac{1}{2}x^{-1/2}\sin^{-1}(x^3) + \frac{3x^{5/2}}{\sqrt{1-x^6}}}$$

(c) $y = \left(\cos^{-1}(4-2x)\right)^5$

$$y' = 5\left(\cos^{-1}(4-2x)\right)^4\left[\cos^{-1}(4-2x)\right]' = 5\left(\cos^{-1}(4-2x)\right)^4 \cdot \left(-\frac{1}{\sqrt{1-(4-2x)^2}}(4-2x)'\right)$$
$$= 5\left[\cos^{-1}(4-2x)\right]^4 \cdot \left(+\frac{1}{\sqrt{1-(4-2x)^2}}(+2)\right)$$
$$= \boxed{\frac{10\left[\cos^{-1}(4-2x)\right]^4}{\sqrt{1-(4-2x)^2}}}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \left| \frac{0}{0} \text{ or } \frac{\infty}{\infty} \right| = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

6. Find the limit.

(a) $\lim_{x\to\infty} \frac{(\ln x)^2}{x-1}$ $= \left| \frac{\infty}{\infty} \right| = \lim_{x\to\infty} \frac{2 \ln x \, \frac{1}{x}}{1} = 2 \lim_{x\to\infty} \frac{\ln x}{x} = \left| \frac{\infty}{\infty} \right|$

$$= 2 \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 2 \lim_{x\to\infty} \frac{1}{x} = \boxed{0}$$

(b) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$ $\left| \frac{0}{0} \right| = \lim_{x\to 0} \frac{\cos x - 1}{3x^2} = \left| \frac{0}{0} \right| = \lim_{x\to 0} \frac{-\sin x}{6x} = \left| \frac{0}{0} \right| = \lim_{x\to 0} \frac{-\cos x}{6} = \boxed{-\frac{1}{6}}$

$\lim_{x\to a} f(x)g(x) = |0\cdot\infty| = \lim_{x\to a} \frac{f(x)}{1/g(x)} = \lim_{x\to a} \frac{g(x)}{1/f(x)}$ (convert the product into the quotient)

(c) $\lim_{x\to 0+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{\ln x}{x^{-2}} \left|\frac{\infty}{\infty}\right|$

$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \left(\frac{x^3}{-2}\cdot\frac{1}{x}\right) = \lim_{x\to 0^+}\left(-\frac{x^2}{2}\right) = \boxed{0}$

$\lim_{x\to a} [f(x)]^{g(x)} = |\infty^0, 0^\infty, 1^\infty| = \lim_{x\to a} e^{g(x)\ln f(x)} = e^{\overbrace{\lim_{x\to a} g(x)\ln f(x)}^{\text{indeterminate product}}}$

(d) $\lim_{x\to\infty} (e^x + x)^{1/x}\ |\infty^0|$

$= \lim_{x\to\infty} e^{\frac{1}{x}\ln(e^x+x)} = e^{\lim_{x\to\infty} \frac{\ln(e^x+x)}{x}} = e^1 = \boxed{e}$

$\lim_{x\to\infty} \frac{\ln(e^x+x)}{x} \left|\frac{\infty}{\infty}\right| = \lim_{x\to\infty} \frac{\frac{1}{e^x+x}(e^x+x)'}{1} = \lim_{x\to\infty} \frac{e^x+1}{e^x+x} \left|\frac{\infty}{\infty}\right|$

$= \lim_{x\to\infty} \frac{e^x}{e^x+1} \left|\frac{\infty}{\infty}\right| = \lim_{x\to\infty} \frac{e^x}{e^x} = 1$

(e) $\lim_{x\to 0}(\sin x)^{\tan x} = \lim_{x\to 0} e^{\tan x(\ln \sin x)} = e^{\lim_{x\to 0} \tan x(\ln \sin x)} = e^0 = \boxed{1}$

$$\lim_{x\to 0} \tan x\left(\ln \sin x\right) = \left|0\cdot\infty\right| = \lim_{x\to 0} \frac{\ln \sin x}{\cot x} \left|\frac{\infty}{\infty}\right|$$

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x}(\cos x)}{-\csc^2 x} = \lim_{x\to 0} \frac{\cos x\left(-\sin^{\cancel{2}} x\right)}{\cancel{\sin x}}$$

$$= (\cos 0)(-\sin 0) = 0$$

convert into the quotient

(f) $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) \left|\infty - \infty\right| = \lim_{x\to 1} \frac{x-1-\ln x}{(x-1)\ln x} \left|\frac{0}{0}\right|$

$$= \lim_{x\to 1} \frac{\left(1-\frac{1}{x}\right)x}{\left(\ln x + \frac{x-1}{x}\right)x} = \lim_{x\to 1} \frac{x-1}{x\ln x + x - 1} \left|\frac{0}{0}\right| = \lim_{x\to 1} \frac{1}{\ln x + x\frac{1}{x} + 1}$$

$$= \lim_{x\to 1} \frac{1}{\ln x + 2} = \frac{1}{\cancel{\ln 1} + 2} = \boxed{\frac{1}{2}}$$

7. The graph of the derivative, $\boxed{f'(x)}$, is shown below. Use the graph to answer these questions.

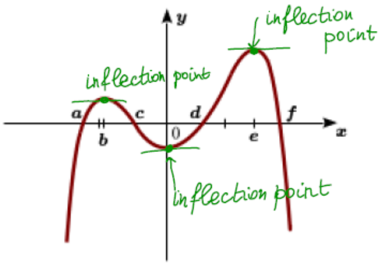
(a) On what intervals is f increasing? decreasing?

(b) On what intervals is f concave up? concave down?

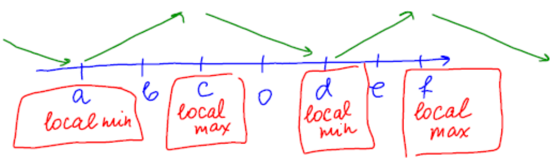
(c) At what values of x does f have a local maximum or minimum?

(d) At what values of x does f have an inflection point?

(e) Assuming that f is continuous and $f(0) = 0$, sketch a graph of f.



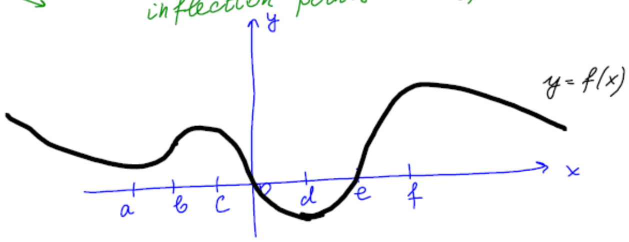
f increases when $f' > 0$	$(a,c) \cup (d,f)$
f decreases when $f' < 0$	$(-\infty,a) \cup (c,d) \cup (f,\infty)$
f is CU when f' is increasing	$(-\infty,b) \cup (0,e)$
f is CD when f' is decreasing	$(b,0) \cup (e,\infty)$



local min: $x = a$, $x = d$

local max: $x = c$, $x = f$

inflection points: $x = b$, $x = 0$, $x = e$



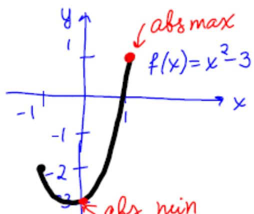
8. Find all absolute and local extrema for the following functions by graphing.

(a) $f(x) = x^2 - 3$, $-1 \le x \le 2$.

$f(-1) = -2$

$f(0) = -3$

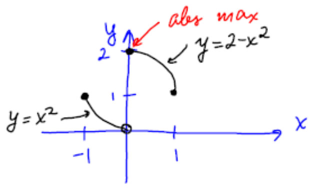
$f(2) = 1$



$\boxed{-3 \text{ is the absolute min}}$

$\boxed{1 \text{ is the absolute max}}$

(b) $f(x) = \begin{cases} x^2, & \text{if } -1 \le x < 0 \\ 2 - x^2, & \text{if } 0 \le x \le 1 \end{cases}$



$\boxed{2 \text{ is the absolute max}}$

$\boxed{\text{no absolute min value}}$

the function is not defined at zero.

c is a critical number if $f'(c)=0$ or $f'(c)$ DNE.

9. Find all critical numbers for the following functions.

(a) $f(x) = \sqrt[3]{x}(x-1)^2$

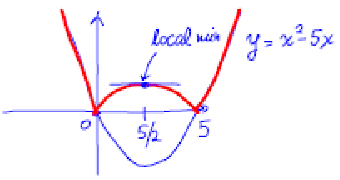
$$f'(x) = \frac{1}{3}x^{-2/3}(x-1)^2 + x^{1/3}\,2(x-1)$$

$$= (x-1)\left[\frac{x-1}{3x^{2/3}} + 2x^{1/3}\right] = (x-1)\,\frac{x-1+2x^{1/3}(3x^{2/3})}{3x^{2/3}}$$

$$= \frac{(x-1)(x-1+6x)}{3x^{2/3}} = \frac{(x-1)(7x-1)}{3x^{2/3}} = 0$$

$$\boxed{x=1,\ x=1/7,\ x=0}$$

(b) $f(x) = |x^2 - 5x| = \begin{cases} x^2 - 5x, & x \le 0 \text{ or } x \ge 5 \\ -(x^2-5x), & 0 < x < 5 \end{cases}$



$f'(x)$ DNE when $\boxed{x=0}$ and $\boxed{x=5}$

$0 < x < 5$, $f(x) = -(x^2-5x)$

$f'(x) = -2x + 5 = 0$

$\boxed{x = \frac{5}{2}}$

(c) $f(x) = xe^{-2x}$

$f'(x) = e^{-2x} + x(-2)e^{-2x}$

$= e^{-2x}(1-2x) = 0$

$1 - 2x = 0$ or $\boxed{x = \frac{1}{2}}$

10. Find the absolute maximum and absolute minimum of the given function on the given interval.

(a) $f(x) = x^3 - 12x + 1$, $[-3, 5]$

Critical numbers:

$$f'(x) = \frac{3x^2 - 12}{3} = \frac{0}{3}$$

$$x^2 - 4 = 0$$

$$x^2 = 4 \quad \text{or} \quad x = \pm 2$$

$$f(-3) = (-3)^3 - 12(-3) + 1 = -27 + 36 + 1 = 10$$

$$f(-2) = (-2)^3 - 12(-2) + 1 = -8 + 24 + 1 = 17$$

$$f(2) = 2^3 - 12(2) + 1 = 8 - 24 + 1 = \boxed{-15 \text{ abs min}}$$

$$f(5) = 5^3 - 12(5) + 1 = 125 - 60 + 1 = \boxed{66 \text{ abs max}}$$

(b) $f(x) = \frac{\ln x}{x}$, $[1, 3]$

$$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 0$$

$1 - \ln x = 0$ or $\ln x = 1$

$x = e$

$$f(1) = \frac{\ln 1}{1} = \boxed{0 \text{ abs min}}$$

$$f(e) = \frac{\ln e}{e} = \boxed{\frac{1}{e}} \text{ abs max} \approx 0.367879$$

$$f(3) = \frac{\ln 3}{3} \approx 0.366$$

(c) $f(t) = 16\cos t + 8\sin 2t$, $\left[0, \frac{\pi}{2}\right]$

$f'(t) = -16\sin t + 8\cos(2t)(2)$

$= \frac{-16\sin t + 16\cos 2t}{16} = \frac{0}{16}$

$-\sin t + \underbrace{\cos 2t}_{1-2\sin^2 t} = 0$

$-\sin t + 1 - 2\sin^2 t = 0$

$2\sin^2 t + \sin t - 1 = 0$

$u = \sin t$, $-1 \leq u \leq 1$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2}$, $u = -1$

$\sin t = \frac{1}{2}$ $\qquad$ $\sin t = -1$

$t = \frac{\pi}{6}$ $\qquad$ $t = \pi$ ← not in the interval

$f(0) = 16\cos 0 + 8\sin 0 = 16$

$f\left(\frac{\pi}{6}\right) = 16\cos\frac{\pi}{6} + 8\sin\frac{2\pi}{6} = 16\frac{\sqrt{3}}{2} + 8\frac{\sqrt{3}}{2} = \frac{24\sqrt{3}}{2} = \boxed{12\sqrt{3} \text{ abs max}}$

$f\left(\frac{\pi}{2}\right) = 16\cos\frac{\pi}{2} + 8\sin\pi = \boxed{0 \text{ abs min}}$