Section 10.2

• Infinite series $\sum_{n=1}^{\infty} a_n$ (n = 1 for convenience, it can be anything).

• Partial sums: $S_N = \sum_{n=1}^{N} a_n$. Note $S_N = S_{N-1} + a_N$.

• If $\{S_N\}_{N=1}^{\infty}$ is convergent and $\lim_{N \to \infty} S_N = S$ exists as a real number, then the series $\sum_{n=1}^{n} a_n$ is convergent. The number $s$ is called the sum of the series.

• Series we can sum:
  - Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$, $-1 < r < 1$
  - Telescoping Series

• THE TEST FOR DIVERGENCE: If $\lim_{n \to \infty} a_n$ does not exist or if $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

• The Test for Divergence cannot be used to prove that a series converges. It can only show a series is divergent.

1. Given a series whose partial sums are given by $s_n = (7n + 3)/(n + 7)$, find the general term $a_n$ of the series and determine if the series converges or diverges. If it converges, find the sum.
2. Find the sum of the following series or show they are divergent:

(a) \[ \sum_{n=1}^{\infty} \frac{7 + 5^n}{10^n} \]

(b) \[ \sum_{n=1}^{\infty} \frac{8}{(n + 1)(n + 3)} \]
3. Write the repeating decimal 0.2\overline{7} as a fraction.

4. Use the test for Divergence to determine whether the series diverges.

(a) \[ \sum_{n=1}^{\infty} \frac{n^5}{3(n^4 + 3)(n + 1)} \]

(b) \[ \sum_{n=1}^{\infty} \arctan n \]

(c) \[ \sum_{n=1}^{\infty} \frac{1}{6 - e^{-n}} \]
Exam 2 Review

1. Evaluate the integral $I = \int (4x^2 - 25)^{-3/2} \, dx$
2. Determine whether the given integral is convergent or divergent.

(a) \[ \int_{1}^{\infty} \frac{4 + \cos^4 x}{x} \, dx \]

(b) \[ \int_{0}^{\infty} \frac{1}{\sqrt{x + e^x}} \, dx \]

(c) \[ \int_{0}^{2016} \frac{1}{\sqrt{2016 - x}} \, dx \]
3. The curve $y = \sin x$ for $0 \leq x \leq \pi$ is rotated about the $x$-axis. Set up, but don’t evaluate the integral for the area of the resulting surface.

4. Determine if the sequence $\{a_n\}_{n=2}^{\infty}$ is decreasing and bounded:
   
   (a) $a_n = \ln n$

   (b) $a_n = \cos(n^2)$

   (c) $a_n = e^{-n}$
(d) \( a_n = e^n + 11 \)

(e) \( a_n = 1 - \frac{1}{n^2} \)

5. The curve \( y = \frac{1}{2} (e^x + e^{-x}) \), \( 0 \leq x \leq 1 \), is rotated about the \( x \)-axis. Find the area of the resulting surface.
6. Set up, \textit{but don't evaluate} the integral for the length of the curve \( x = 2t^2, \quad y = t^3, \quad 0 \leq t \leq 1. \)

7. Find length of the curve \( y = \frac{1}{\pi} \ln(\sec(\pi x)) \) from the point \((0, 0)\) to the point \((\frac{1}{6}, \ln \frac{2}{\sqrt{3}})\).
8. Use a trigonometric substitution to eliminate the root: \( \sqrt{24 - 12x + 2x^2} \).

9. Determine if the sequence converges or diverges. If converges, find its limit.

(a) \( \left\{ \frac{2016 + (-1)^n}{n^{2016}} \right\}_{n=1}^{\infty} \)

(b) \( \left\{ \sqrt{\frac{7n + 6n^3 + n^2}{(n + 3)(n^2 + 8)}} \right\}_{n=4}^{\infty} \)

(c) \( \left\{ \frac{1}{2} \ln(n^2 + 2n - 4) - \ln(n + 6) \right\}_{n=10}^{\infty} \)
10. Evaluate the integral \[ \int \frac{(x - 1)^2}{5\sqrt{25 - (x - 1)^2}} \, dx. \]
11. Compute \( S = \sum_{n=1}^{\infty} \left( e^{1/n} - e^{1/(n+1)} \right) \).

12. Write out the form of the partial fraction decomposition (do not try to solve)

\[
\frac{20x^3 + 12x^2 + x}{(x^3 - x)(x^3 + 2x^2 - 3x)(x^2 + x + 1)(x^2 + 9)^2}
\]
13. Evaluate the integral \( \int \frac{5x^2 + x + 12}{x^3 + 4x} \, dx \)
14. Assuming that the sequence defined recursively by $a_1 = 1$, $a_{n+1} = \frac{1}{2} \left( a_n + \frac{16}{a_n} \right)$ is convergent, find its limit.

15. For what values of $x$ the series $\sum_{n=0}^{\infty} (4x - 3)^{n+3}$ converges? What is the sum of the series?