

MATH 308 Homework 3

- 3.1. Find the solution of the given initial value problem

$$ty' + (t + 1)y = t, \quad y(\ln 2) = 1, \quad t > 0.$$

- 3.2. Find the general solution of the differential equation $2y' + y = 3t^2$ and determine how solutions behave as $t \rightarrow \infty$.

- 3.3. Solve the initial value problem

$$2y' - y = e^{t/3}, \quad y(0) = a.$$

Describe the behavior of the solution depending on a . What is the value of a for which there a transition from one type of behavior to another occurs.

- 3.4. Consider the initial value problem

$$y' + \frac{1}{2}y = 2 \cos t, \quad y(0) = -1$$

and find the coordinates of the first local maximum point of the solution for $t > 0$.

- 3.5. Find the solution of the given initial value problem

$$y' = xy^3(1 + x^2)^{-1/2}, \quad y(0) = 1,$$

determine the interval in which the solution is defined.

- 3.6. Sketch the graph of the function $y^2(y^2 - 1)$ versus y , determine the equilibrium point of the equation

$$y' = y^2(y^2 - 1).$$

Which of them are stable?

- 3.7. Determine whether the equation

$$(2xy^2 + 2y) + (2x^2y + 2x)y' = 0.$$

If it is exact, then solve the equation.

- 3.8. Find the value of b for which the given equation is exact, and then solve it using that vlaue of b .

$$(ye^{2xy} + x) dx + bxe^{2xy} dy = 0.$$

- 3.9. Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}.$$