

Math 308 — Problem Set 7
Issued: 10.29 Due: Training Homework

7.1. Find the general solution of the system

$$\begin{cases} x_1' = 3x_1 - 2x_2 \\ x_2' = 2x_1 - 2x_2 \end{cases}$$

7.2. Solve the given initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

Describe the behavior of the solution as $t \rightarrow \infty$.

7.3. Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}.$$

7.4. Solve the given initial value problem.

$$\mathbf{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ -3 \end{pmatrix}.$$

Describe the behavior of the solution as $t \rightarrow \infty$.

7.5. Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}.$$

7.6. Find the general solution of the equation

$$y'' + 2y' - 8y = 0.$$

- 7.7.** Use the method of reduction of order to find a second solution y_2 of the given differential equation such that $\{y_1, y_2\}$ is a fundamental set of solutions.

$$xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2.$$

- 7.8.** Find a general solution of the given Cauchy-Euler equation for $x > 0$.

$$x^2y'' + 4xy' + 2y = 0.$$

- 7.9.** A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/s, and if there is no damping, determine the position y of the mass at any time t .

- 7.10.** Find the general solution of the equation

$$2y'' + 3y' + y = t^2 + 3 \sin t.$$