A counterexample for finite element projectors

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In one space dimension, $L_2$-orthogonal projectors onto spline spaces are known to be uniformly bounded as operators in $L_\infty$ or $C$, independently of the choice of the partition. After results by Ciesielski and de Boor for low-degree splines obtained about 40 years ago, this was recently established in full generality by Shadrin [1].

In higher dimensions, things are "easier". It was folklore since the 70ies that uniform $L_\infty$ bounds for $L_2$-orthogonal projectors onto multivariate spline spaces are not to be expected. However, even for the simplest case of linear $C^0$ finite element spaces over triangulations of polygonal domains, concrete proof was not given in the literature. In [2], we provided such an example. It shows that in the 2D case the $L_\infty$ norm may grow at least linearly with the number of triangles in the partition (certainly, these "bad" triangulations cannot satisfy the minimum angle condition uniformly in the number of triangles). A matching upper bound has not yet been proved.

References