Homework #1. (Due Jan. 22)
Math. 417

These are just from the book. I realize that not everyone has been able to obtain the book yet.

Problem 1. (#1, P.14) Show that the following equations have at leas one solution in the given intervals.

(a) $x\cos x - 2x^2 + 3x - 1 = 0$. [.2,.3] and [1.2,1.3].
(b) $(x-2)^2 - \ln x = 0$, [1,2] and [e,4].
(c) $2x\cos(2x) - (x-2)^2 = 0$, [2,3] and [3,4]
(d) $x - (\ln\ x)^x = 0$, [4,5]

Problem 2. (#7, P.14) Let $f(x) = x^3$.

(a) Find the second Taylor polynomal $P_2(x)$ about $x_0 = 0$.
(b) Find $R_2(.5)$ and the actual error using $P_2(.5)$ to approximate $f(.5)$.
(c) Repeat part(a) using $x_0 = 1$.
(d) Repeat part (b) using the polynomial from part (c).

Problem 3. (#9, p.14) Find the second Taylor polynomial $P_2(x)$ for the functions $f(x) = e^x \cos x$ about $x_0 = 0$.

(a) Use $P_2(.5)$ to approximation f(.5). Find an upper bound for the error $|f(.5) - p_2(.5)|$ using the error formula and compare it to the actual error.
(b) Find a bound for the error $|f(x) - P_2(x)|$ using $P_2(x)$ to approximate $f(x)$ on the interval $[0, 1]$.
(c) Approximate $\int_0^1 f(x)\,dx$ using $\int_0^1 P_2(x)dx$.
(d) Find an upper bound for the error in (c) using (b) and compare to the actual error.

Problem 4. (#26, p.16) Suppose that $f \in C[a, b]$, x_1 and x_2 are in $[a, b]$, and c_1 and c_2 are positive constants. Show that a number ζ exists between x_1 and x_2 with

$$f(\zeta) = \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2}.$$

(see hint on the assignment page).