Homework #6. (Due March 6.)

Math. 417

Problem 1. *(a) Write an algorithm which preprocesses the denominators*

$$d_i = \prod_{j=0, j\neq i}^{n} (x_i - x_j)$$

appearing in the Lagrange form of the interpolating polynomial based on the nodes $\{x_0, x_1, \ldots, x_n\}$. *Your algorithm should involve* $O(n^2)$ *operations.*

(b) Write an algorithm which computes

$$P(x) = \sum_{i=0}^{n} l_i(x) y_i$$

given the values of the denominators (above) and x. *Your algorithm should involve* $O(n)$ *operations. (The algorithm I am thinking of also requires* $O(n)$ *tests).*

Problem 2. *Given distinct nodes* $\{x_0, x_1, \ldots, x_n\}$ *and* $\{y_0, y_1, \ldots, y_n\}$, *consider writing the interpolating polynomial*

$$P_n(x) = \sum_{i=0}^{n} c_i x^i$$

where the coefficients $\{c_0, c_1, \ldots, c_n\}$ *are unknowns. There are* $n+1$ *unknowns* $\{c_0, c_1, \ldots, c_n\}$ *and* $n+1$ *equations,*

$$P_n(x_i) = y_i, \qquad i = 0, 1, \ldots, n.$$

As the x_i *are fixed, this problem can be written as a matrix vector problem*

$$Ac = y.$$

Here A *is an* $(n+1) \times (n+1)$ *matrix depending on the nodes.*

(a) What are the entries of the matrix A*?*
(b) Is the matrix A *nonsingular (why or why not)?*
(c) What would be the work required to solve this system by Gaussian elimination?