

# A Two-Grid Goal-Oriented Iterative Solver for $hp$ -FE Discretizations with Possibly Elongated Elements of Elliptic Problems

D. Pardo, L. Demkowicz, and C. Torres-Verdín  
The University of Texas at Austin, Austin TX, USA

## ABSTRACT

A three-dimensional self-adaptive goal-oriented  $hp$ -Finite Element Method has been developed at The University of Texas at Austin. This adaptive strategy iterates along the following steps. Given an arbitrary  $hp$ -grid, we first construct the corresponding  $h/2, p+1$ -grid, that is, each element is divided into eight new elements, and the polynomial order of approximations is uniformly increased by one. Then, we utilize the solution over the  $h/2, p+1$  grid to determine optimal refinements over our original  $hp$ -grid. Following these optimal refinements, we construct our next optimal grid that we utilize as the starting point for the next iteration.

To efficiently compute the solution of the problem over the  $h/2, p+1$ -grid becomes crucial to guarantee the applicability of this method to practical problems. Furthermore, for a number of electromagnetic logging applications, elongated elements and/or Perfectly Matched Layers (PML's) typically appear as a result of the geometry or any other particular feature of the problem.

The main objective of the work presented in here is to study, design, and implement an efficient solver for arbitrary  $h/2, p+1$ -grids with possibly elongated elements and/or PML's for elliptic problems.

Driven by the adaptive strategy, we shall utilize a two-grid solver, with the coarse and fine grids given by the  $hp$ -grid and  $h/2, p+1$  grids, respectively. We shall analyze the performance of different smoothers as well as practical implementation aspects such as the convenience or not of assembling stiffness matrices and/or prolongation operator matrices.

We shall finish the talk by applying the two-grid solver to a number of electrostatic logging applications of interest.

For additional info and references, please visit [www.ices.utexas.edu/~pardo](http://www.ices.utexas.edu/~pardo)

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