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MATH 151 Honors Final Exam Fall 2005

Sections 201,202 P. Yasskin

Multiple Choice: (4 points each)

1. Compute $\lim_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x^2}$

a. -1

b. 0

c. $\dfrac{1}{2}$

d. 1

e. ∞

2. Compute $\lim_{x\to 3^-} \dfrac{x}{x^2-9}$

a. $-\infty$

b. $-\dfrac{1}{3}$

c. 0

d. $\dfrac{1}{3}$

e. ∞

3. Compute $\lim_{n\to\infty} \dfrac{3}{n^3}\dfrac{n(n+1)(2n+1)}{6}$

a. 0

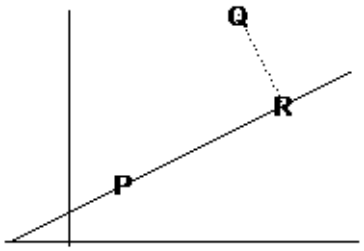
b. $\dfrac{1}{2}$

c. 1

d. 2

e. 3

4. If you drop a perpendicular from the point $Q = (3,4)$ to the line through $P = (1,1)$ with direction $\vec{v} = (2,1)$, then the foot of the perpendicular is $R =$



a. $(3.2, 2.0)$

b. $(3.4, 2.2)$

c. $(3.6, 2.3)$

d. $(3.8, 2.4)$

e. $(4.0, 2.5)$

5. On which interval is there a solution of the equation $x^5 - x^4 + x^3 - x^2 + x - 1 = -4$?

a. $[-2, -1]$

b. $[-1, 0]$

c. $[0, 1]$

d. $[1, 2]$

e. $[2, 3]$

6. If $g(x) = 3x^2 e^{(x^3+1)}$ then $g'(x) =$

a. $18x^3 e^{(x^3+1)}$

b. $6xe^{(3x^2)}$

c. $6xe^{(x^3+1)} + 3x^2 e^{(x^3+1)}$

d. $(6x + 9x^4)e^{(x^3+1)}$

e. $(6x + 9x^4 + 3x^2)e^{(x^3+1)}$

7. A rocket starts at rest on the ground and has vertical acceleration $a(t) = \frac{\pi^2}{900}\sin\left(\frac{\pi t}{30}\right)$ where distance is in km and time is in sec. Find the rocket's altitude after 30 sec.

a. $\frac{\pi}{30}$ km

b. $\frac{30}{\pi}$ km

c. $\frac{1}{\pi}$ km

d. 30 km

e. π km

8. For an ideal gas, the pressure, P, volume, V, and temperature, T, are related by $PV = kT$ where k is a constant. For a certain sample of gas the current values are

$$P = 100 \text{ kPa} \qquad V = 5 \text{ m}^3 \qquad T = 250°\text{K} \qquad \text{and consequently} \qquad k = 2\ \frac{\text{kPa} \cdot \text{m}^3}{°\text{K}}$$

If the volume and temperature are increasing at

$$\frac{dV}{dt} = 0.2\ \frac{\text{m}^3}{\text{sec}} \qquad \text{and} \qquad \frac{dT}{dt} = 6\ \frac{°\text{K}}{\text{sec}}$$

is the pressure increasing or decreasing and at what rate?

a. decreasing at $1.6\ \frac{\text{kPa}}{\text{sec}}$

b. decreasing at $8\ \frac{\text{kPa}}{\text{sec}}$

c. increasing at $1.6\ \frac{\text{kPa}}{\text{sec}}$

d. increasing at $8\ \frac{\text{kPa}}{\text{sec}}$

e. Pressure is constant.

9. The right half of the hyperbola $x^2 - y^2 = 1$ may be parametrized by

$$x = \cosh(t) = \frac{e^t + e^{-t}}{2}, \qquad y = \sinh(t) = \frac{e^t - e^{-t}}{2}$$

Find the y-intercept of the tangent line at $t = \ln 2$.

NOTE: $\cosh(\ln 2) = \dfrac{2 + 1/2}{2} = \dfrac{5}{4}$ $\sinh(\ln 2) = \dfrac{2 - 1/2}{2} = \dfrac{3}{4}$

a. $-\frac{4}{5}$

b. $\frac{4}{5}$

c. $-\frac{17}{10}$

d. $\frac{17}{6}$

e. $-\frac{4}{3}$

10. For the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 2$ the point at $x = 1$ is a

a. local minimum.

b. local maximum.

c. horizontal inflection point.

d. non-horizontal inflection point.

e. Cannot be determined because the Second Derivative Test Fails.

11. Find the area under $y = 2x - x^2$ above the x-axis.

a. $\frac{2}{3}$

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. $\frac{16}{3}$

e. $\frac{32}{3}$

12. Compute $\int_0^2 xe^{x^2} dx$

a. $\frac{1}{2}e^2 - \frac{1}{2}$

b. $2e^2$

c. $2e^4 - 2$

d. $\frac{1}{2}e^4 - \frac{1}{2}$

e. $2e^4$

Work Out: (Points indicated. Part credit possible.)

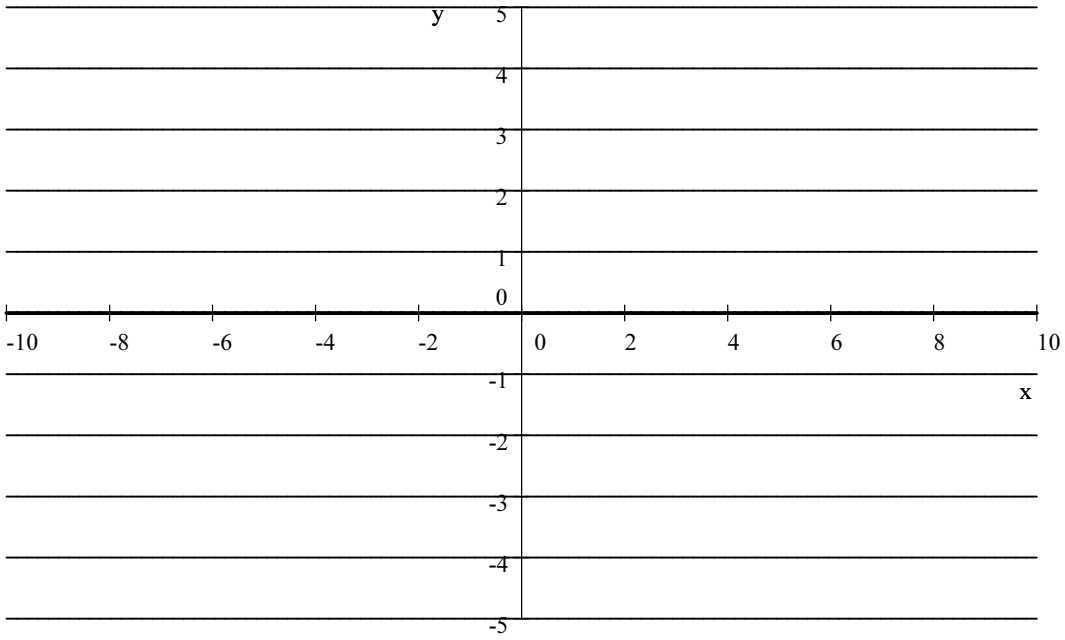
13. (12 points) Plot a function which satisfies

$f(-3) = -2$ $\quad f(-1) = -3$ $\quad f(0) = 0$ $\quad f(1) = 2$

$\lim_{x \to 2^-} f(x) = -\infty$ $\quad \lim_{x \to 2^+} f(x) = \infty$ $\quad \lim_{x \to -\infty} f(x) = -1$ $\quad \lim_{x \to \infty} f(x) = 1$

$f'(x) > 0$ on $(-1, 1)$ $\quad f'(x) < 0$ on $(-\infty, -1) \cup (1, 2) \cup (2, \infty)$

$f''(x) > 0$ on $(-3, 0) \cup (2, \infty)$ $\quad f''(x) < 0$ on $(-\infty, -3) \cup (0, 2)$



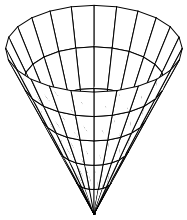
14. (9 points) Consider the function $f(x) = x^3 + px$. (Explain your reasoning.)

a. For what values of p is $f(x)$ strictly decreasing on the interval $[-1, 3]$?

b. For what values of p is $f(x)$ strictly increasing on the interval $[-1, 3]$?

c. For what values of p is $f(x)$ concave up everywhere on the interval $[-1, 3]$?

15. (12 points) You want to make a paper drinking cup in the shape of a cone. If the cup is to hold 81π cm^3, what are the radius, r, and height, h, of the cone which uses the least paper? Why?



HINTS: The volume and area of a cone are

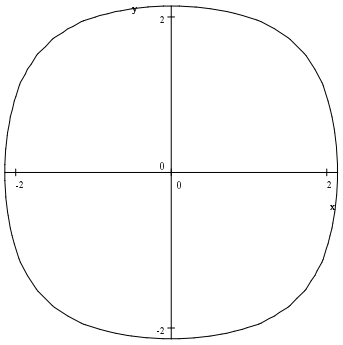
$$V = \frac{1}{3}\pi r^2 h \qquad\qquad A = \pi r\sqrt{r^2 + h^2}$$

16. (11 points) Let $V(x) = xe^x$ and let $W(x)$ be the inverse function of $V(x)$. Thus,

$$y = W(x) \quad \text{means} \quad x = V(y) = ye^y$$

Find an expression for $W'(x)$, the derivative of $W(x)$.
(Your answer may involve x and $W(x)$ but not y.
If possible, also eliminate any exponentials.)

17. (12 points) The graph of the equation $x^4 + x^2y^2 + y^4 = 21$ is shown at the right.



a. Find the equation of the line tangent to the graph at the point $(1,2)$.
HINT: Find the slope by implicit differentiation.

b. The graph implicitly defines a function $y = f(x)$ which passes through the point $(1,2)$.
Use the linear approximation to $f(x)$ at $x = 1,$ to estimate $f(1.3)$.