

Name (Print) _____ ID _____

Last, First Middle

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MATH 152

FINAL EXAM

Spring 2000

Sections 513,514

P. Yasskin

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13	/13
14	/13
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Multiple Choice: (4 points each)

1. Compute $\int_0^{\pi/2} x \cos(3x) dx$

- a. $-\frac{\pi}{3} - \frac{1}{3}$
- b. $-\frac{\pi}{6} - \frac{1}{3}$
- c. $-\frac{\pi}{3} + \frac{1}{3}$
- d. $-\frac{\pi}{6} - \frac{1}{9}$
- e. $-\frac{\pi}{3} + \frac{1}{9}$

2. Compute $\lim_{n \rightarrow \infty} \frac{2^n}{1 + 3^n}$

- a. 0
- b. $\frac{1}{2}$
- c. $\frac{1}{1 - \frac{2}{3}}$
- d. $\frac{\frac{1}{2}}{1 - \frac{2}{3}}$
- e. ∞

3. Compute $\int_0^{\pi/2} \sin^3 \theta d\theta$

- a. $-\frac{2}{3}$
- b. $-\frac{1}{3}$
- c. 0
- d. $\frac{1}{3}$
- e. $\frac{2}{3}$

4. Which formula will give the arclength of the curve $y = \sin x$ between $x = 0$ and $x = \pi$?

a. $L = \int_0^{\pi} 2\pi x \sqrt{1 + \cos^2 x} dx$

b. $L = \int_0^{\pi} \sqrt{1 + \cos^2 x} dx$

c. $L = \int_0^{\pi} 2\pi \sin x \sqrt{1 + \cos^2 x} dx$

d. $L = \int_0^{\pi} 2\pi x \sqrt{1 + \sin^2 x} dx$

e. $L = \int_0^{\pi} \sqrt{1 + \sin^2 x} dx$

5. Which initial value problem describes the solution to the following problem:

A 100 gal tank is initially filled with sugar water whose concentration is $0.05 \frac{\text{lb sugar}}{\text{gal water}}$. Sugar is added to the tank at the rate of $2 \frac{\text{lb}}{\text{hr}}$ and pure water is added at the rate of $3 \frac{\text{gal}}{\text{hr}}$. The mixture is kept well mixed and drained at the rate of $3 \frac{\text{gal}}{\text{hr}}$. Find the amount of sugar in the tank after t hours.

a. $\frac{dS}{dt} = 2 - 0.03S, \quad S(0) = 5$

b. $\frac{dS}{dt} = 0.1 - 0.15S, \quad S(0) = 5$

c. $\frac{dS}{dt} = 3S - 0.02, \quad S(0) = 0.05$

d. $\frac{dS}{dt} = 0.02 - 3S, \quad S(0) = 5$

e. $\frac{dS}{dt} = 0.02 - 0.03S, \quad S(0) = 0.05$

6. Find the solution of the differential equation $\frac{dy}{dx} = 2x(1 + y^2)$ satisfying the initial condition $y(2) = 0$.

a. $y = \tan(x^2) + 2$

b. $y = \tan^2(x - 2)$

c. $y = \tan(x^2 - 4)$

d. $y = \tan(x^2 + \arctan 2)$

e. $y = \tan^2(x) - \tan^2 2$

7. Compute $\int_1^2 \frac{1}{(x-2)^{2/3}} dx$

- a. $-\infty$
- b. -3
- c. -1
- d. 3
- e. ∞

8. Compute $\lim_{x \rightarrow 0} \frac{\sin(2x) - 2x}{3x^3}$

- a. $-\frac{1}{9}$
- b. -4
- c. $-\frac{4}{9}$
- d. $-\frac{8}{9}$
- e. $-\frac{4}{3}$

9. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)^2} (x-3)^n$.

- a. 0
- b. $\frac{1}{2}$
- c. 2
- d. $\frac{1}{3}$
- e. 3

10. Which term is incorrect in the following partial fraction expansion?

$$\frac{-10x^2 + 5x^3 - 8x + 1}{(x-1)(x-3)^2(x^2+2)} = \underbrace{\frac{A}{x-1}}_{\text{a.}} + \underbrace{\frac{B}{x-3}}_{\text{b.}} + \underbrace{\frac{D}{(x-3)^2}}_{\text{c.}} + \underbrace{\frac{Ex+F}{x^2+2}}_{\text{d.}}$$

e. They are all correct.

11. A vector \vec{u} has length 3. A vector \vec{v} has length 4. The angle between them is 60° . Find $\vec{u} \cdot \vec{v}$.

- a. 6
- b. $\frac{1}{24}$
- c. $\frac{\sqrt{3}}{24}$
- d. 24
- e. $6\sqrt{3}$

12. Find an equation for the plane containing the two lines

$$\begin{aligned} L_1 : & \quad x = 3 + 3t & y = 1 + 4t & z = 2 + 5t \\ L_2 : & \quad x = 3 + t & y = 1 & z = 2 - t \end{aligned}$$

- a. $-4x - 8y - 4z = 10$
- b. $-4x + 8y - 4z = 10$
- c. $x - 2y + z = 3$
- d. $x + 2y + z = 7$
- e. $x + 2y + z = 10$

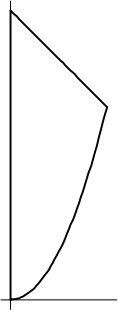
Work Out (13 points each)

Show all your work. Partial credit will be given. You may not use a calculator.

13. Compute $\int \frac{\sqrt{x^2 - 1}}{x} dx$

14. The parametric curve given by $x = t^2$, $y = \frac{2}{3}t^3$, $z = \frac{1}{4}t^4$ for $0 \leq t \leq 2$ is rotated about the y -axis. Find the area of the surface of revolution.
HINT: Factor the quantity in the square root.

15. The region in the first quadrant between the curves $y = x^2$ and $y = 6 - x$ is rotated about the y -axis. Find the volume of the solid of revolution.



16. A water tank has the shape of a circular cylinder laying on its side. It is 3 ft in radius and 5 ft long. It is half full of water. How much work is needed to pump the water out a spout at the top? (The weight density of water is $\rho g = 64.5 \frac{\text{lb}}{\text{ft}^3}$ but you may leave your answer as a multiple of ρg .)

