

Student (Print) \_\_\_\_\_

Last, First Middle

Section \_\_\_\_\_

Student (Sign) \_\_\_\_\_

Student ID \_\_\_\_\_

Instructor \_\_\_\_\_

MATH 152  
Exam2  
Fall 2001  
Test Form A

Part I is multiple choice. There is no partial credit.

You may not use a calculator.

Part II is work out. Show all your work. Partial credit will be given.

You may use a calculator which cannot hold formulas.

1-10	/50
11	/10
12	/6
13	/6
14	/8
15	/10
16	/10
TOTAL	

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator.

1. Compute  $\int_2^3 \frac{1}{(x-2)^{4/3}} dx$

- a. The integral diverges.
- b.  $-1$
- c.  $1$
- d.  $-3$
- e.  $3$

2. For which values of  $p$  does the integral  $\int_1^{\infty} x^{-p} dx$  converge?

- a. Only  $p = 1$
- b. All values of  $p > 1$
- c. All values of  $p < 1$
- d. All values of  $p$
- e. The integral diverges for all values of  $p$

3. Compute the length of the arc  $y = \frac{2}{3}x^{3/2}$  for  $0 \leq x \leq 3$ .

- a.  $7$
- b.  $14$
- c.  $\frac{14}{3}$
- d.  $\frac{16}{3}$
- e.  $\frac{\sqrt{7}}{3}$

4. Find the  $x$ -coordinate of the center of mass of a plate with constant density  $\rho$  bounded by the curves

$$x = 0, \quad y = 0 \quad \text{and} \quad y = 8 - x^3$$

- a.  $\frac{48}{5}\rho$
  - b.  $12\rho$
  - c.  $\frac{5}{48}$
  - d.  $\frac{4}{5}$
  - e.  $\frac{5}{4}$
5. If  $y = f(x)$  is the solution to the differential equation  $12\frac{dy}{dx} = xy^2$  which passes through the point  $(0,1)$ , what is  $f(4)$ ?

- a.  $\frac{-1}{4}$
- b.  $\frac{6}{5}$
- c. 3
- d.  $\frac{12}{11}$
- e. 5

6. What is the integrating factor for the differential equation  $\frac{dy}{dx} = 4xy + 5$ ?

- a.  $\frac{1}{x^4}$
- b.  $x^4$
- c.  $e^{-4x}$
- d.  $e^{2x^2}$
- e.  $e^{-2x^2}$

7. Compute the limit of the sequence  $a_n = \ln(2n + 1) - \ln(n)$  for  $n = 1, 2, \dots$

- a.  $-\ln 2$
- b. 0
- c. 1
- d.  $\ln 2$
- e.  $\infty$

8. Let  $I = \int_a^b f(x) dx$  and let  $T_n$  be the Trapezoid rule approximation of  $I$  with  $n$  intervals. If  $f''(x) > 0$  for all  $x$ , which of the following is true?
- $T_n < I$
  - $T_n > I$
  - $T_n > I$  for some values of  $n$  and  $T_n < I$  for other values of  $n$
  - $T_n = I$
  - $T_n = I$  for large values of  $n$

9. Which of the following integrals gives the area of the surface of revolution obtained by rotating the curve

$$x(t) = 3t - t^3, \quad y(t) = 3t^2 \quad \text{for } 0 \leq t \leq 1$$

about the  $x$ -axis?

- $6\pi \int_0^1 (3t - t^3)(t^2 + 1) dt$
  - $18\pi \int_0^1 (t^2 + t^4) dt$
  - $2\pi \int_0^1 \frac{3t^2}{\sqrt{3 - 3t^2}} dt$
  - $2\pi \int_0^1 \frac{3t - t^3}{\sqrt{3 - 3t}} dt$
  - $2\pi \int_0^1 \frac{3t^2}{\sqrt{(3t - t^3)^2 + (3t^2)^2}} dt$
10. The sequence  $a_n = \frac{(-1)^n}{n^2 + 2}$ , for  $n = 0, 1, 2, \dots$  is
- decreasing and divergent.
  - increasing and divergent.
  - increasing and convergent.
  - non-monotonic and divergent.
  - non-monotonic and convergent.

Part II: Work Out

Show all your work. Partial credit will be given.

You may use a calculator which cannot hold formulas.

11. (10 points) Solve the initial value problem

$$x \frac{dy}{dx} - 3y = x^4 \quad \text{for } x > 0 \quad \text{with } y(1) = 0.$$

12. (6 points)

- a. (3 pts) What is wrong with the following calculation?

$$\int_{-1}^1 x^{-6} dx = \left[ \frac{x^{-5}}{-5} \right]_{-1}^1 = -\frac{2}{5}$$

- b. (3 pts) What is the value of  $\int_{-1}^1 x^{-6} dx$ ?

13. (6 points) Use the Comparison Test to determine if  $\int_1^{\infty} \frac{dx}{\sqrt{x^4 + x}}$  converges or diverges.

You must clearly state (i) the integral you are using for comparison, (ii) whether it converges or diverges (You don't need to prove this.) and (iii) why this implies the original integral converges or diverges.

14. (8 points) What is the area of the surface generated when the arc of the circle  $x^2 + y^2 = 100$  for  $5 \leq x \leq 6$  is revolved about the  $x$ -axis?

15. (10 points)

a. (2 pts) Write a definite integral for the arclength of the parabola  $y = \frac{x^2}{2}$  for  $0 \leq x \leq 1$ .

(Your answer should be  $L = \int_0^1 \sqrt{1+x^2} dx$  but be sure to show your work.)

b. (4 pts) Use the trapezoid rule with  $n = 5$  intervals to approximate this arclength.

(Use your calculator to evaluate your answer. Keep 5 decimal places.)

c. (1 pt) The arclength is  $L = \int_0^1 g(x) dx$  where  $g(x) = \sqrt{1+x^2}$ . Show  $g''(x) = \frac{1}{(1+x^2)^{3/2}}$ .

d. (3 pts) Compute a bound on the error in the trapezoid rule approximation found in part (b) by using the trapezoid error bound formula

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{where } K \geq |g''(x)| \text{ for } a \leq x \leq b$$

16. (10 points) A brine solution that contains 0.3 kg of salt per liter flows at a constant rate of  $20 \frac{\text{L}}{\text{min}}$  into a large tank that initially held 1000 L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at the constant rate of  $20 \frac{\text{L}}{\text{min}}$ . Let  $S(t)$  be the amount of salt in the tank at time  $t$ .
- Set up the differential equation and initial condition for  $S(t)$ .

- How much salt is in the tank after one hour?