

Name _____ Sec _____

(Print) Last, First Middle

Signature _____ ID _____

MATH 152 Final Exam Fall 2001

Sections 510,511 P. Yasskin

Multiple Choice: (4 points each)

| | |
|------|-----|
| 1-13 | /52 |
| 14 | /12 |
| 15 | /12 |
| 16 | /12 |
| 17 | /12 |

1. Which of the following gives the trapezoid approximation to $\int_1^9 e^{-x^2} dx$ with 4 intervals?

- a. $e^{-1} + e^{-9} + e^{-25} + e^{-49} + e^{-81}$
- b. $\frac{1}{2}e^{-1} + e^{-9} + e^{-25} + e^{-49} + \frac{1}{2}e^{-81}$
- c. $e^{-1} + 2e^{-9} + 2e^{-25} + 2e^{-49} + e^{-81}$
- d. $2e^{-1} + 2e^{-9} + 2e^{-25} + 2e^{-49} + 2e^{-81}$
- e. $2e^{-1} + 4e^{-9} + 4e^{-25} + 4e^{-49} + 2e^{-81}$

2. Compute $\int_1^3 \frac{x}{(x^2 - 1)^{2/3}} dx$

- a. $-\infty$
- b. $\frac{3}{2}(\sqrt[3]{3} - 1)$
- c. $\frac{3}{2}\sqrt[3]{2}$
- d. 3
- e. ∞

3. Compute $\lim_{x \rightarrow 0} \frac{x \sin(x^2) - \sin(x^3)}{x^7}$

- a. $-\frac{1}{6}$
- b. $-\frac{1}{2}$
- c. 0
- d. $\frac{1}{2}$
- e. $\frac{1}{6}$

4. The region below $y = e^{-x}$ in the first quadrant is rotated about the x -axis. Find the volume of the solid of revolution.
- $\frac{\pi}{4}$
 - $\frac{\pi}{2}$
 - π
 - 2π
 - 4π
5. A 20 meter chain is hanging down the side of a building. Its mass density is $\rho = 6 + 6\sin^2(\pi x) \frac{\text{kg}}{\text{m}}$ where x is measured in meters down from the top of the building. Find the total mass of the chain.
- 60 kg
 - 90 kg
 - 120 kg
 - 150 kg
 - 180 kg
6. Find the total area between $y = 4x$ and $y = x^3$.
- 0
 - 2
 - 4
 - 8
 - 16
7. Find an equation of the line perpendicular to the plane $-2x + 3y + z = 7$ which contains the point $P = (3, 2, 1)$
- $x = 3 - 2t$ $y = 2 + 3t$ $z = 1 + t$
 - $x = 3 - 2t$ $y = 2 - 3t$ $z = 1 + t$
 - $x = -2 + 3t$ $y = 3 + 2t$ $z = 1 + t$
 - $x = -2 + 3t$ $y = -3 + 2t$ $z = 1 + t$
 - $x = -2 + 3t$ $y = 3 - 2t$ $z = 1 + t$

8. The Maclaurin series $\sum_{n=1}^{\infty} \frac{(-x)^{n+1}}{n!}$ converges to

- a. xe^{-x}
- b. $-xe^{-x}$
- c. $\frac{1-e^{-x}}{x}$
- d. $\frac{e^{-x}-1}{x}$
- e. $x - xe^{-x}$

9. Find the area of the triangle whose **edges** are

$$\vec{u} = (2, -1, 2), \quad \vec{v} = (0, 1, 2) \quad \text{and} \quad \vec{w} = (2, -2, 0).$$

- a. $\frac{1}{2}\sqrt{6}$
- b. 3
- c. 6
- d. 18
- e. 36

10. The substitution $x = (\tan \theta)^2$ turns the integral $\int_0^1 \frac{dx}{(1+x)\sqrt{x}}$ into

- a. $\int_0^{\pi/4} 2 d\theta$
- b. $\int_0^{\pi/4} 2 \sec \theta d\theta$
- c. $\int_0^{\pi/2} 2 d\theta$
- d. $\int_0^{\pi/2} \frac{d\theta}{2 \sec \theta}$
- e. $\int_0^{\pi/2} \frac{2 d\theta}{\sec \theta}$

11. Compute $\int_{-1}^2 \frac{5}{x^2 - x - 6} dx$

- a. 0
- b. $-2\ln 4$
- c. $\ln 4$
- d. $2\ln 4$
- e. ∞

12. The function $y(x)$ satisfies the differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1+x^2}$ and the initial condition $y(0) = 0$. What is $y(1)$?

- a. $\frac{\pi}{4}$
- b. $\frac{\pi}{2}$
- c. $\frac{1}{2}$
- d. $\frac{1}{\sqrt{2}}$
- e. 1

13. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{4^n}$.

- a. $\frac{1}{4}$
- b. $\frac{1}{2}$
- c. 2
- d. 3
- e. 4

Work Out (12 points each)

Show all your work. Partial credit will be given. You may not use a calculator.

14. Compute $\int_0^{\pi/2} (x - \pi) \sin x dx$

15. The curve $x = t^2$, $y = \frac{1}{3}t^3 - t$ for $0 \leq t \leq 1$ is rotated about the y -axis. Find the area of the surface of revolution.

16. A 150 lb iron ball is hanging down the side of a building at the bottom of a 20 ft chain which weighs $5 \frac{\text{lb}}{\text{ft}}$. What is the total work done to lift the ball and chain to the top of the building?

17. Determine if each of the following series converges or diverges. Say why.
Be sure to name or quote the test(s) you use and check out all requirements of the test.

a.
$$\sum_{n=0}^{\infty} \frac{n^2(-5)^n}{(n+2)!}$$

Explain:

Circle one: Converges Diverges

b.
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Explain:

Circle one: Converges Diverges

c.
$$\sum_{n=2}^{\infty} (-1)^n \frac{2n+1}{n^2+3}$$

Explain:

Circle one: Converges Diverges