

Symbolic and Numerical Integration in MATLAB

1 Symbolic Integration in MATLAB

Certain functions can be symbolically integrated in MATLAB with the *int* command.

Example 1. Find an antiderivative for the function

$$f(x) = x^2.$$

We can do this in (at least) three different ways. The shortest is:

```
>>int('x^2')
ans =
1/3*x^3
```

Alternatively, we can define x symbolically first, and then leave off the single quotes in the *int* statement.

```
>>syms x
>>int(x^2)
ans =
1/3*x^3
```

Finally, we can first define f as an inline function, and then integrate the inline function.

```
>>syms x
>>f=inline('x^2')
f =
Inline function:
>>f(x) = x^2
>>int(f(x))
ans =
1/3*x^3
```

In certain calculations, it is useful to define the antiderivative as an inline function. Given that the preceding lines of code have already been typed, we can accomplish this with the following commands:

```
>>intoff=int(f(x))
intoff =
1/3*x^3
>>intoff=inline(char(intoff))
intoff =
Inline function:
intoff(x) = 1/3*x^3
```

The inline function $intoff(x)$ has now been defined as the antiderivative of $f(x) = x^2$. \triangle
The int command can also be used with limits of integration.

Example 2. Evaluate the integral

$$\int_1^2 x \cos x dx.$$

In this case, we will only use the first method from Example 1, though the other two methods will work as well. We have

```
>>int('x*cos(x)',1,2)
ans =
cos(2)+2*sin(2)-cos(1)-sin(1)
>>eval(ans)
ans =
0.0207
```

Notice that since MATLAB is working symbolically here the answer it gives is in terms of the sine and cosine of 1 and 2 radians. In order to force MATLAB to evaluate this, we have to use the $eval$ command. \triangle

For many functions, the antiderivative cannot be written down in a closed form (as the sum of a finite number of terms), and so the int command cannot give a result. As an example, the function

$$f(x) = e^{-x^2}$$

falls into this category of functions. If we try int on this function, we get:

```
int('exp(-x^2)')
ans =
1/2*pi^(1/2)*erf(x)
```

where by $erf(x)$ MATLAB is referring to the function

$$erf(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy,$$

which is to say, MATLAB hasn't actually told us anything. In cases like this, we can proceed by evaluating the integral numerically.

2 Numerical Integration in MATLAB

MATLAB has two primary tools for the numerical evaluation of integrals of real-valued functions, the $quad$ command which uses an adaptive Simpson's method (we will discuss Simpson's method in the next section) and the $quadl$ command which uses the an adaptive Lobatto method (we probably won't discuss the Lobatto method).

Example 3. Evaluate the integral

$$\int_1^2 e^{-x^2} dx.$$

We use

```
quad('exp(-x.^2)',1,2)
ans =
0.1353
```

The *quad* command requires an input function that can be appropriately evaluated for vector values of the argument, and so we have used an array operation. \triangle

The *quad* command can also be used in order to evaluate functions defined in M-files. In this way it's possible to integrate functions that have no convenient closed form expression.

Example 4. Evaluate the integral

$$\int_0^{10} y(x) dx,$$

where y is implicitly defined by the relationship

$$x = y^3 + e^y.$$

In this case, we cannot solve explicitly for y as a function of x , and so we will write an M-file that takes values of x as input and returns the associated values of y as output.

```
function value = yfunction(x)
syms y;
f=inline('x-y^3-exp(y)','x','y');
for k=1:length(x)
value(k) = fzero(@(y) f(x(k),y), .5);
end
```

The *for* loop is necessary so that the function *yfunction* can be evaluated at vector values for the independent variable x , as required by the *quad* command. We find

```
quad(@yfunction,0,10)
ans =
9.9943
```

(There is also an alternative approach to this type of problem that involves relating the integral of $y(x)$ to the integral of $x(y)$, but that's not the topic of this section.)

3 Assignments

1. Find an antiderivative for the function

$$f(x) = x \sin^2 x.$$

2. Evaluate the integral

$$\int_1^2 x \sin^2 x dx.$$

3. Evaluate the integral

$$\int_1^2 \sin(x^2) dx.$$

4. Evaluate the integral

$$\int_0^2 y(x) dx,$$

where y is defined implicitly by the relation

$$x = y + \sin y.$$