## M401 Spring 2010, Assignment 1, due Thursday January 28

1. [10 pts] In many introductory textbooks on ODE, air resistance is modeled with a linear term (because this gives rise to equations that are easy to solve). In this case the equation for a falling object is

$$y'' = -g - by',$$

where $y(t)$ denotes the height of the object at time $t$, $g$ denotes gravitational acceleration at the earth's surface, and $b$ is the coefficient of air resistance (or, more precisely, $b = \frac{B}{m}$, where $B$ is the coefficient of air resistance). Determine the dimensions of $b$ and nondimensionalize this equation. Specify your initial conditions, assuming the original initial conditions are $y(0) = h$ and $y'(0) = v$.

2. [10 pts] Consider an object of mass $m$ moving along a frictionless surface and attached to a wall by a spring. If we let $y(t)$ denote the spring's displacement from equilibrium, and we assume $y(t)$ is small, then the motion can often be modeled by an equation of the form

$$my'' = -k_1 y + k_3 y^3,$$

where $k_1$ is Hooke's constant and the term $k_3 y^3$ is a nonlinear correction corresponding with the fact that the spring will weaken if either strongly stretched or strongly compressed. Find the dimensions of $k_1$ and $k_3$ and nondimensionalize this equation.

3. [10 pts] Exercise 1.14 on p. 24 of Simmonds and Mann Jr.

**Notes and suggestions.** Begin by specifying the dimensions of $EI$ (combined) and $k$. (You don't need to know anything about the physics to do this, though see my solutions for a brief discussion.) I suggest replacing the letter $T$ with $F$ (for force) since tension is a force and we will typically use $T$ to denote the dimension time. Either at the beginning or (as I recommend) at a convenient later stage you'll need to divide by a constant with dimensions of $p$ to make each term dimensionless. Be clear in the end about your definitions of $\epsilon$, $\beta$, and $f$.

4. [10 pts] Exercise 1.4 on p. 12 of Simmonds and Mann Jr.

**Notes and suggestions.** Use Taylor's Theorem (instead of the expansion method). Before solving this problem, show that the Implicit Function Theorem will apply in this case and explain what Taylor's Theorem and the Implicit Function Theorem guarantee about the existence and accuracy of your approximation.

5. [10 pts] Find power series approximations for the roots of

$$x^3 + \epsilon x^2 - x + \epsilon = 0,$$

with an error of order $\epsilon^3$. Follow the same notes and suggestions as in Problem 4.