M401 Spring 2010, Assignment 11, due Thursday April 29

1a. [5 pts] Compute the Fourier cosine series for $f(x) = \sin x$ on $[0, \pi]$.

1b. [3 pts] Find an upper bound on the error obtained if the first N terms of the Fourier cosine series from Part (a) are used as an approximation for $f(x) = \sin x$, where N denotes a positive *even* integer.

1c. [2 pts] Explain how your result from Part (b) ensures that there cannot be a Gibbs Phenomenon for this series.

2a. [5 pts] Compute the Fourier sine series for $f(x) = \cos x$ on $[0, \pi]$.

2b. [3 pts] Explain why our method of error estimation fails for this Fourier sine series.

2c. [2 pts] Explain why there *is* a Gibbs Phenomenon for this series, and specify the values $x \in [0, \pi]$ where it occurs.

3. [10 pts] Exercise 5.6 in Constanda, Parts (i) and (iv).

4. [10 pts] Solve the wave equation

$$\begin{aligned} u_{tt} &= c^2 u_{xx} \\ u_x(0,t) &= 0; \quad u(L,t) = 0; \quad t \geq 0 \\ u(x,0) &= f(x); \quad x \in [0,L] \\ u_t(x,0) &= g(x); \quad x \in [0,L]. \end{aligned}$$

5. [10 pts] Solve the inhomogeneous heat equation

$$\begin{aligned} u_t &= k u_{xx} + e^x + 1 \\ u(0,t) &= 1; \quad u(L,t) = 5; \quad t \geq 0 \\ u(x,0) &= f(x); \quad x \in [0,L]. \end{aligned}$$