

M401 Spring 2010, Assignment 5, due Thursday Feb. 25

1. [10 pts] We saw in Problem 5 from Assignment 3 that for

$$\begin{aligned}y'' + k^2y &= \epsilon y^2 \\ y(0) &= 1 \\ y'(0) &= 0,\end{aligned}$$

the function $y_1(t)$ in the perturbation expansion $y(t) = y_0(t) + \epsilon y_1(t) + \dots$ does not have a secular term to remove. It turns out that the function $y_2(t)$ does have a secular term. Find a Poincare expansion for the solution to this equation, with terms $y(t) \approx y_0 + \epsilon y_1$ and $\lambda(\epsilon) \approx k + \lambda_1\epsilon + \lambda_2\epsilon^2$. (You will have to write down the y_2 equation in order to find λ_2 , but you won't have to solve it.) Be clear about the size of the error for your approximation, and the interval of time for which it is valid. If you combine this calculation with that of Problem 5 from Assignment 3 then you've solved Exercise 4.3 from p. 55 of Simmonds and Mann Jr.

2. [10 pts] Try applying Poincare's method to the van der Pol equation

$$\begin{aligned}y'' + \epsilon y'(y^2 - 1) + y &= 0 \\ y(0) &= 1 \\ y'(0) &= 0,\end{aligned}$$

and explain where it fails and why.

3. [10 pts]

3a. Find all values a for which the difficulty you had with Poincare's method in problem 2 does not arise for the van der Pol equation

$$\begin{aligned}y'' + \epsilon y'(y^2 - 1) + y &= 0 \\ y(0) &= a \\ y'(0) &= 0.\end{aligned}$$

(Notice that the initial condition has changed.)

3b. Choose any positive value of a from your list in (3a) and find the Poincare solution of the van der Pol equation with terms $y(t) \approx y_0 + \epsilon y_1$ and $\lambda(\epsilon) \approx k + \lambda_1\epsilon + \lambda_2\epsilon^2$.

4. [10 pts] Find the first order two-scale approximation for

$$\begin{aligned}y'' + \epsilon y^4 y' + k^2 y &= 0 \\ y(0) &= 1 \\ y'(0) &= 0.\end{aligned}$$

Be clear about the size of the error for your approximation, and the interval of time for which it is valid. **Hint.** Use $\cos^4 \theta \sin \theta = \frac{1}{8} \sin \theta + \frac{3}{16} \sin 3\theta + \frac{1}{16} \sin 5\theta$.

5. [10 pts] Find the first order two-scale approximation to the van der Pol equation

$$\begin{aligned}y'' + \epsilon y'(y^2 - 1) + y &= 0 \\ y(0) &= 1 \\ y'(0) &= 0.\end{aligned}$$

Be clear about the size of the error for your approximation, and the interval of time for which it is valid. **Hint.** The ODE

$$\begin{aligned}C_1' &= \frac{1}{2}C_1 - \frac{1}{8}C_1^3 \\ C_1(0) &= 1,\end{aligned}$$

is fairly easy to solve with separation of variables and partial fractions. You can skip this calculation and use the result,

$$C_1(\tau) = \frac{2}{\sqrt{1 + 3e^{-\tau}}}.$$