

M401 Spring 2010, Assignment 7, due Thursday April 1

1a. [5 pts] Non-dimensionalize the wave equation

$$\begin{aligned}u_{tt} &= c^2 u_{xx}; & (x, t) &\in (0, L) \times (0, \infty) \\u(0, t) &= U_1; & u(L, t) &= U_2; & t > 0 \\u(x, 0) &= f(x); & x &\in [0, L] \\u_t(x, 0) &= g(x), & x &\in [0, L].\end{aligned}$$

1b. [5 pts] Non-dimensionalize the Navier-Stokes momentum equation

$$\begin{aligned}\rho(u_t + uu_x) &= \mu u_{xx} - p_x + f(x, t); & (x, t) &\in (0, L) \times (0, \infty) \\u(0, t) &= U_1; & u(L, t) &= U_2; & t > 0 \\u(x, 0) &= g(x); & x &\in [0, L].\end{aligned}$$

Take both f and p to be given functions. (Though p is typically an unknown, and this equation is typically coupled with the continuity equation to give a system of two equations for the unknowns u and p .) Begin by specifying the dimensions of the viscosity coefficient μ . Your non-dimensionalized equation should have the form

$$v_\tau + vv_\xi = \frac{1}{R} v_{\xi\xi} - \phi_\xi + g(\xi, \tau),$$

where R denotes the *Reynold's number*

$$R = \frac{LU_1\rho}{\mu}.$$

Note. The Reynold's number is an important dimensionless constant in the theory of fluids. In the case of three space dimensions, the values L and U_1 are typically chosen to correspond with the geometry of the particular problem.

2. The coating of surfaces by thin fluid films is a critical industrial process that arises in applications such as the protection of microchips, de-icing of airplane wings, and the construction of photographic film. Consider, for example, painting a wall. What you would like to do is simply brush a single thick line of paint across the top of the wall and let the paint descend in a steady sheet to the floor. Unfortunately, in most circumstances the paint drips down in *fingers*, and the wall is not smoothly covered. Through the use of mathematical modeling, we can create laboratory situations in which the paint descends as a steady wall. In this problem, we will take the first step toward such a model by deriving a partial differential equation for the height $h(t, x)$ of a thin film moving along a surface (see Figure 1). Assuming the film has constant density ρ , that it is moving with velocity $V(t, x)$ in the x -direction only, and that it is uniform in the y -direction (same height and velocity for all y over a steady width $y \in [0, L]$), derive a partial differential equation that takes a given $V(t, x)$ and describes the height $h(t, x)$. Discuss the types of initial values and boundary values your model would require, and what they mean physically.

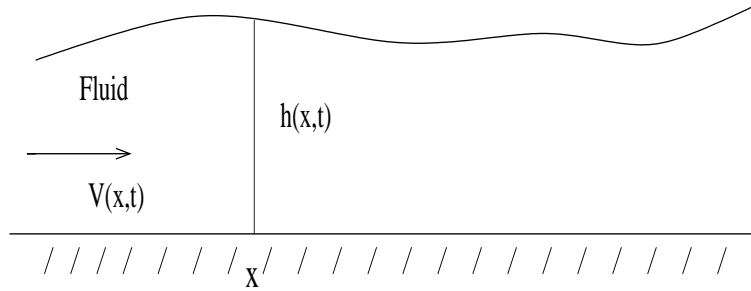


Figure 1: Flow of a thin film.

3. Constanda Exercise 1, Parts (i) and (ii).

Note. Constanda gives the answers to his exercises, so feel free to check your work, but also be sure that your solution is justified.

4. Constanda Exercise 1, Parts (iii) and (iv).

5. Constanda Exercise 2, Parts (i) and (ii).