

M401 Spring 2010, Assignment 8, due Thursday April 8

1. [10 pts] Constanda Exercise 12.3, Parts (i) and (ii).
2. [10 pts] Constanda Exercise 12.4, Parts (i) and (ii).
3. [10 pts] Constanda Exercise 12.5, Parts (i) and (ii).
4. [10 pts] Constanda Exercise 12.6, Parts (i) and (ii). Explain why the solution to Part (ii) is continuous, while the solution to Part (i) is not. For Part (ii) show that your solution is equivalent to Constanda's.
- 5a. [4 pts] Solve the PDE

$$\begin{aligned}u_{tt} &= c^2 u_{xx}; & (x, t) &\in (0, \infty) \times (0, \infty) \\u(0, t) &= h(t), & t &\geq 0 \\u(x, 0) &= 0, & x &\geq 0 \\u_t(x, 0) &= 0, & x &\geq 0.\end{aligned}$$

For consistency, assume $h(0) = 0$.

- 5b. [3 pts] Use your solution from (5a) to solve the specific problem

$$\begin{aligned}u_{tt} &= 4u_{xx}; & (x, t) &\in (0, \infty) \times (0, \infty) \\u(0, t) &= \sin(\pi t), & t &\geq 0 \\u(x, 0) &= 0, & x &\geq 0 \\u_t(x, 0) &= 0, & x &\geq 0.\end{aligned}$$

Sketch plots of $u(x, \frac{1}{2})$ and $u(x, 1)$.

- 5c. [3 pts] Use your solution from (5a), our calculations from class, and the method of superposition to write down a solution to the PDE

$$\begin{aligned}u_{tt} &= c^2 u_{xx}; & (x, t) &\in (0, \infty) \times (0, \infty) \\u(0, t) &= h(t), & t &\geq 0 \\u(x, 0) &= f(x), & x &\geq 0 \\u_t(x, 0) &= g(x), & x &\geq 0.\end{aligned}$$

For consistency, assume $h(0) = f(0)$.