M412 Assignment 1, due Friday, September 2

A prerequisite for M412 is M308, Differential Equations. These problems constitute a brief review of the some of the material covered in M308.

1. [5 pts] Solve the initial value problem

$$\frac{dy}{dx} = 3x^2(y^2 + 1); \quad y(0) = 1.$$

For what domain is your solution valid?

2. [5 pts] Find a general solution for the differential equation

$$(x^2 + 1)\frac{dy}{dx} + 3xy = 6x.$$

3. [5 pts] Show that the boundary value problem

$$y''(x) + 3y(x) = 0; \quad y(0) = 0, \quad y(\pi) = 0$$

has only the trivial solution $y(x) \equiv 0$.

4. [5 pts] Show that the boundary value problem

$$y''(x) + 4y(x) = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

has an infinite number of solutions.

5. [10 pts] Consider the boundary value problem

$$y''(x) + \lambda y(x) = 0; \quad y(0) = 0, \quad y(\pi) = 0, \tag{1}$$

for some parameter λ. If there exists a nontrivial solution to (1) for some value of λ, then we say that λ is an eigenvalue of (1). We refer to the corresponding solution as the eigenfunction associated with λ. Notice that we saw in Problem 3 that $\lambda = 3$ is not an eigenvalue, while we saw in Problem 4 that $\lambda = 4$ is an eigenvalue. For equation (1), find the *real* values of λ that are eigenvalues and determine their associated eigenfunctions.