M442 Assignment 4, due Friday Oct. 11

1. [10 pts] In this problem we will compare fits obtained by transforming an equation to linear form and using linear regression versus fits obtained directly from nonlinear regression. The Malthusian model for population growth is

\[ \frac{dy}{dt} = ry; \quad y(0) = y_0, \]

with exact solution \( y(t) = y_0 e^{rt} \). (As in our analysis in class of the logistic equation we will regard \( y_0 \) as a parameter.)

a. Using the U.S. population data in uspop.m (available on the course web site), find regression values for \( y_0 \) and \( r \) using the linear relationship

\[ \ln y = \ln y_0 + rt. \]

(Notice that MATLAB uses \( \log \) for natural logarithm \( \ln \).) Plot your transformed data along with your regression line, and also plot the curve \( y(t) = y_0 e^{rt} \) along with the original data. Compute the error

\[ E(r, y_0) = \sum_{k=1}^{23} (y_k - y_0 e^{rt_k})^2. \]

b. Use lsqcurvefit.m to fit the data in uspop.m directly to the nonlinear expression \( y(t) = y_0 e^{rt} \). Plot your fit along with the data in this case, and compare your result with your result from (a). Also, compare your values of \( E(r, y_0) \).

2. [10 pts] Suppose we have data \( \{(t_k, y_k)\}_{k=1}^{N} \) for which the independent variables are equally spaced, with

\[ h = t_{k+1} - t_k \]

for \( k = 1, 2, \ldots, N - 1 \). Show that

\[ y'(t_k) = \frac{y(t_k + h) - y(t_k - h)}{2h} + O(h^2), \]

for \( k = 2, 2, \ldots, N - 1 \).

3. [10 pts] The Gompertz model for population growth is described through the ODE

\[ \frac{dy}{dt} = -ry \ln \left( \frac{y}{K} \right); \quad y(0) = y_0, \]

with exact solution

\[ y(t; r, K, y_0) = K \left( \frac{y_0}{K} \right) e^{-rt}. \]

a. Write the Gompertz ODE in a linear form and use this form and the U.S. population data in uspop.m (available on the course web site) to obtain rough estimates for values of the parameters \( r, K, \) and \( y_0 \). (Notice that as with our analysis of the logistic model in class, we will treat \( y_0 \) as a parameter. You can use either forward differences or central differences to approximate the derivative.)

b. Use lsqcurvefit to obtain nonlinear regression values for \( r, K, \) and \( y_0 \). Plot your fit along with the data. Which model better describes U.S. population growth, logistic or Gompertz?