M442 Assignment 5, due Friday Oct. 23

1. [5 pts] For the system of differential equations

$$\begin{aligned} x' &= 1 + ax^2 - by^2 \\ y' &= by + axy, \end{aligned}$$

suppose the point $(x_e, y_e) = (1, 2)$ is known to be an equilibrium point. Use this to find values for the parameters a and b.

2. [10 pts] For the Lotka–Volterra model

$$\begin{aligned} \frac{dy_1}{dt} &= ay_1 - by_1y_2 \\ \frac{dy_2}{dt} &= -ry_2 + cy_1y_2, \end{aligned}$$

write down the phase plane equation and solve it to obtain an implicit relationship between y_1 and y_2. For the parameter values $a = .4732$, $b = .0240$, $c = .0234$, and $r = .7646$, use MATLAB to sketch the integral curve associated with the initial values $y_1(0) = 30$, $y_2(0) = 4$. (I suggest *ezplot* for the plotting; see Section 3.5 in the course notes MATLAB Basics.)

3. [5 pts] Compute the Jacobian of each of the following functions:

3a.

$$f(\vec{y}) = y_1^2 + y_2 - y_1e^{y_3}.$$

3b.

$$\vec{f}(\vec{y}) = \begin{pmatrix} y_1y_2^2 \\ y_2 \sin y_3 \end{pmatrix}.$$

4. [10 pts] Consider the first order system of ordinary differential equations

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -x + x^2. \end{aligned}$$

Find all equilibrium points for this system and sketch the phase plane diagram near each. Determine whether or not each is stable.

5. [5 pts] For the ODE system

$$\begin{aligned} \frac{dy_1}{dt} &= y_1y_2^2 + y_1y_3^2 \\ \frac{dy_2}{dt} &= y_1 - y_2 + y_3 \\ \frac{dy_3}{dt} &= (1 - y_2)(1 + y_3), \end{aligned}$$

find all equilibrium points and write down the linearized system associated with each.

6. [10 pts] Use the second characterization (involving ϵ) in our definition of differentiability to prove the following useful chain rule: If $\vec{g} : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $\vec{x}_0$ and $\vec{f} : \mathbb{R}^m \to \mathbb{R}^l$ is differentiable at $\vec{g}(\vec{x}_0)$ then

$$D_x \vec{f}(\vec{g}(\vec{x}_0)) = \vec{f}'(\vec{g}(\vec{x}_0))\vec{g}'(\vec{x}_0).$$

Here, by D_x we mean differentiation with respect to $\vec{x}$ rather than the argument of $\vec{f}$. (**Hint.** This is actually fairly easy, especially for such an important result. Begin by writing down what if means for g to be differentiable at $\vec{x}_0$ and for $\vec{f}$ to be differentiable at $\vec{g}(\vec{x}_0)$. Then specify clearly what it is you're trying to show.)