Marble Descent through a Viscous Fluid

M442, Fall 2009

Due Friday, October 9

Overview

In this project, we consider an experiment in which marbles are dropped through a viscous fluid and the terminal velocities of the marbles are recorded. Our goal will be to use dimensional analysis and regression to determine a model for these terminal velocities as a function of the given parameters. (NOTE: While it's possible[1] using Stokes' Law, Archimedes' Principle, and Newton's second law of motion to model this phenomenon with ordinary differential equations, that is not the goal of this project.)

Since the viscosity of a fluid is critical to this project, I'll discuss it briefly here. Roughly speaking, viscosity measures the resistance to motion due to collisions between particles. Imagine yourself as a particle walking across campus: density is a measure of how many people are walking around, and viscosity is a measure of how many times they're stopping to talk. In this way a campus crowded with people who do not know one another would have a high density and a low viscosity, while a campus with a few people who know each other well would have a low density and a high viscosity.[2]

The viscosity of a fluid is measured by a determination of the amount of resistance the fluid poses to motion through it. One fairly easy way to accomplish this is with Stokes' Law

$$F = 6\pi\mu v r,$$

which describes the resistance force F on a sphere with radius r moving with velocity v through a fluid with viscosity μ (the sphere is assumed to be reasonably far from the boundary).[3] Stokes' Law assumes *laminar* flow, by which we mean fluid particles move along

[1]In fact, pretty easy.

[2]Unless the people don't like each other, but you see what I'm getting at.

[3]Strictly speaking, this is not the definition of viscosity. For the standard definition, consider an infinitesimally thin plate with area A moving with velocity V through a viscous fluid, parallel to a flat surface, and separated from the flat surface by a distance h. (Something like a frisbee over a soccer field, the frisbee a height h from the ground.) Since viscosity sets up a drag between the fixed surface and the thin plate, if the force on the plate is held constant, the velocity V of the plate will depend on h. In particular, for a *Newtonian fluid*, V will increase linearly as h increases ($V = \alpha h$ for some constant α). Technically, the viscosity of a fluid is defined as that constant value μ such that $F = \mu A \frac{V}{h}$ (often written $F = \mu A \frac{dV}{dh}$ or $F = \mu \alpha A$), where F is the constant force acting to push the plate with velocity V, or alternatively can be thought of as the force due to resistance on the plate. (The relation $F = \mu A \frac{V}{h}$ is true, by definition, for a *Newtonian fluid*, a class which includes all gases and most common liquids including water and oil. For non-Newtonian fluids (e. g., mud, milk, blood) the viscosity changes with changing force, and consequently we can't talk about *the* viscosity of the fluid.)

smooth paths in thin layers (lamina). In order to obtain a value for μ, we incorporate F into a model of the sphere (based on standard Newtonian mechanics) and estimate μ from data. More precisely, the common metric unit of *absolute* viscosity is the *poise*, which is defined as the force in dynes[4] required to move a surface that is one square centimeter in area past a parallel surface at a speed of one centimeter per second, with the surfaces separated by a fluid film one centimeter thick (i.e., the units are dyne $\cdot$ s/cm^2, with associated dimensions $\mathrm{ML}^{-1}\mathrm{T}^{-1}$). The SI unit for viscosity is actually pascal-seconds, but this is seldom used in practice. Of course, all this is for a fixed temperature, and viscosity changes dramatically with temperature.

Experimental Data

Data was collected for two fluids and five marbles. Physical properties of the fluids and marbles are listed in Table 1, while terminal velocities are tabulated in Table 2.

- **Marble 1.** Mass $m = .003$ kg, radius $r = .0070$ m.
- **Marble 2.** Mass $m = .005$ kg, radius $r = .0080$ m.
- **Marble 3.** Mass $m = .013$ kg, radius $r = .0110$ m.
- **Marble 4.** Mass $m = .022$ kg, radius $r = .0130$ m.
- **Marble 5.** Mass $m = .054$ kg, radius $r = .0175$ m.
- **Fluid 1**. Kroger's Light Corn syrup, Density $\rho = 1427.1$ kg/m^3,Viscosity $\mu = 5.0$ kg/m/s.
- **Fluid 2**. Suave Daily Clarifying Shampoo, Density $\rho = 1018.0$ kg/m^3,Viscosity $\mu = 20.0$ kg/m/s.

Table 1: Physical properties of fluids and marbles.

Fluid/Marble	Marble 1	Marble 2	Marble 3	Marble 4	Marble 5
Corn syrup	.026 m/s	.030 m/s	.046 m/s	.070 m/s	.076 m/s
Shampoo	.009 m/s	.010 m/s	.016 m/s	.019 m/s	.021 m/s

Table 2: Experimental terminal velocities.

[4] One dyne is the amount of force required to give a free mass of one gram an acceleration of one centimeter per second per second.

Assignments

1. Use dimensional analysis to determine a general form for the terminal velocity of a marble descending through a viscous fluid. (The first step here consists of deciding what the key variables are; of course in this regard the data I've given you is a pretty good hint.) Your report should contain a typeset version of this analysis.

2. Use the data in Tables 1 and 2 to develop a predictive model of the terminal velocity for any marble and fluid. Your report should contain a tabulation of dimensionless products used for this calculation and any plots you used in the fitting.

3. Write a MATLAB function M-file that takes as input the appropriate variables, as determined in Assignment 1, and returns the terminal velocity associated with a particular marble moving through a particular fluid. In class on Friday Oct. 9, you will test your model by predicting the terminal velocity for the marble and fluid presented in class. You will write your prediction at the top of your report, and your project grade will be based entirely on its accuracy.[5]

[5] Just kidding.