M611, Fall 2008, Assignment 6

Due Friday, Oct. 10

1. [10 pts] Evans 2.5.2.

Note. Recall that we refer to an $n \times n$ matrix O as orthogonal provided $O^{tr} = O^{-1}$.

2. [10 pts] Recall from our discussion of physicality that the gravitational (force) field $\vec{\mathcal{F}}$ corresponding with mass distributed in space with density $\rho(\vec{x})$ is

$$\vec{\mathcal{F}} = -D\phi,$$

where the potential ϕ solves Poisson's equation

$$-\Delta\phi = -4\pi G\rho(\vec{x}),$$

where G is Newton's gravitational constant. Use this formulation to derive an expression for the gravitational field $\vec{\mathcal{F}}$ associated with mass uniformly distributed over a ball $B(0, R) \subset \mathbb{R}^3$ with constant density ρ. Be sure to consider points both inside and outside $B(0, R)$. (**Note.** I suggest placing $\vec{x}$ on the vertical axis (without loss of generality) and using the law of cosines with the polar angle ϕ.)

3. [10 pts] Evans 2.5.3.

4. [10 pts] Evans 2.5.4.

5. [10 pts] Evans 2.5.5.

Note. One approach is to consider $v(\vec{x}) = M_g + M_f(e^2 - e^{x_1+1})$, where

$$M_g := \max_{\vec{x}\in\partial B(0,1)} |g(\vec{x})|$$
$$M_f := \max_{\vec{x}\in B(0,1)} |f(\vec{x})|.$$