M611 Fall 2014, Assignment 8, due Friday Nov. 7

1. [10 pts] Find the Green’s function for Laplace’s equation on the quarter plane \( U = \mathbb{R}_+ \times \mathbb{R}_+ \). Use your Green’s function to solve Laplace’s equation

\[
\Delta u = 0 \quad \text{in } U,
\]
\[
u = g\quad \text{on } \partial U.
\]

You need not prove that \( u(\vec{x}) \) defined this way is a solution.

2. [10 pts] Find the Green’s function for Laplace’s equation on the infinite wedge

\[
U = \{ (r, \theta) : 0 < r < \infty, 0 < \theta < \frac{\pi}{3} \}.
\]

In this case you only need to find \( G(\vec{x}, \vec{y}) \); in particular, you do not need to use it to write down a solution to Laplace’s equation.

3. [10 pts] (Evans 2.5.7.) Use Poisson’s formula for the ball to prove

\[
r^{n-2} \frac{r - |\vec{x}|}{(r + |\vec{x}|)^{n-1}} u(0) \leq u(\vec{x}) \leq r^{n-2} \frac{r + |\vec{x}|}{(r - |\vec{x}|)^{n-1}} u(0)
\]

whenever \( u \) is positive and harmonic in \( B^o(0, r) \). This is an explicit form of Harnack’s inequality.

4. [10 pts] (Evans 2.5.8.) Prove Theorem 15 of Section 2.2.4. (Hint: Since \( u \equiv 1 \) solves (44) for \( g \equiv 1 \), the theory automatically implies

\[
\int_{\partial B(0,1)} K(\vec{x}, \vec{y}) dS_y = 1
\]

for each \( \vec{x} \in B^o(0, 1) \).)