

Final Exam Topics

The final exam for M611 will be in the usual classroom EPH 206, Wednesday, Dec. 10, 10:30 a.m.-12:30 p.m. The exam will cover the course material that was not covered on the midterm exam; in particular, distribution theory, the heat equation, the wave equation, and the method of characteristics. The exam will consist of five or six questions, worth five to ten points each. Two of the questions will ask for explicit calculations and the others will ask for proofs. I will provide statements of the system of characteristic equations and of the d'Alembert, Poisson, and Kirchhoff solutions to the wave equation. You will need to bring your own paper.

1. Explicit calculations

There will be two questions on the exam involving explicit calculations. The basic areas from which I'll draw these questions are as follows:

- Computing the derivative of a distribution (see Problems 4 and 5 from Assignments 8-9)
- Solutions obtained formally by operator methods (see Problem 1 from Assignment 12)
- Solutions of the wave equation on $\mathbb{R} \times \mathbb{R}_+$ (see Problems 1 and 2 from Assignment 11)
- Solutions of first order linear, quasilinear and nonlinear PDE by the method of characteristics (see Problems 2 and 3 from Assignment 12)

2. Proofs

Three or four questions on the exam will ask you to prove an assertion using techniques discussed in the course. The basic areas from which I'll draw these questions are as follows:

- Prove that a given mapping is a distribution (see Problem 3 from Assignments 8-9)
- Verify properties of distributions (see Problems 6-9 from Assignments 8-9)
- Verify properties of the heat kernel and properties of solutions to the heat equation on $\mathbb{R}^n \times \mathbb{R}_+$ (see Problem 1 from Assignment 10)
- Verify properties of the heat equation on U_T (see Evans 2.5.14, though I wouldn't expect you to be as familiar with the proof of Theorem 2.3.3 as this problem assumes)
- Derive properties of solutions of the wave equation, especially (though not only) in cases in which the properties can be obtained from the d'Alembert, Poisson, and Kirchhoff integral representations (see Problems 2.5.17 and 2.5.18 from Evans)

- Know the properties of the straightening transformation $\vec{\Phi}$ and its role in our development of the Method of Characteristics.
- Aspects of our development of the Method of Characteristics, especially regarding our process of bringing everything back to the characteristic equations. For example, show that if $\vec{p}(s; \tilde{y})$, $z(s; \tilde{y})$, and $\vec{x}(s; \tilde{y})$ solve the characteristic equations with admissible initial data $(\vec{q}(\tilde{y}), g(\tilde{y}), \tilde{y})$ for $(s; \tilde{y}) \in I \times W$ then

$$F(\vec{p}(s; \tilde{y}), z(s; \tilde{y}), \vec{x}(s; \tilde{y})) = 0$$

for all $(s, \tilde{y}) \in I \times W$.