M641, Fall 2012, Assignment 4, due Wed. Sept. 26

1 [10 pts]. (**Keener Problem 1.2.2.**)

a. Prove that two symmetric matrices are equivalent if and only if they have the same eigenvalues with the same multiplicities.

b. Show that if A and B are equivalent then $\det A = \det B$.

c. Is the converse of (b) true?

2 [10 pts]. Prove that if the eigenvalues of a self-adjoint matrix are all positive the matrix must be positive definite.

Note. We say a self-adjoint matrix is positive definite if $\langle A\vec{z}, \vec{z}\rangle > 0$ for all $\vec{z} \in \mathbb{C}^n$, $\vec{z} \neq 0$.

3 [10 pts]. (**Keener Problem 1.2.3.**)

a. Show that if A is an $n \times m$ matrix and B is an $m \times n$ matrix, then AB and BA have the same nonzero eigenvalues.

b. Show that for any $n \times m$ matrix A the eigenvalues of AA^* are real and non-negative.

4 [10 pts]. Use the Maximum Principle (Keener's Theorem 1.6) to state and prove an analogous *Minimum Principle.*

5 [10 pts]. (**Keener Problem 1.3.1.**) Use the minimax principle to show that the matrix

$$A = \begin{pmatrix} 2 & 4 & 5 & 1 \\ 4 & 2 & 1 & 3 \\ 5 & 1 & 60 & 12 \\ 1 & 3 & 12 & 48 \end{pmatrix}$$

has an eigenvalue $\lambda_4 < -2.1$ and an eigenvalue $\lambda_1 > 67.4$.