

1 Review: Bayes' Theorem

(Example from Lial, Greenwell, and Ritchey)

Example 1.1 *In the 1994 elections for the U.S. House of Representatives, 46% of men and 54% of women voted Democratic. Of those who voted, 49% were men and 51% were women. What is the probability that a person who voted Democratic was a man?*

(Source: New Your Times, Nov 13, 1994)

Example 1.2 *We have two tanks of fish. In the first tank are 3 white fish and 2 orange fish. In the second tank are 4 white and 1 orange fish. A fish is moved from the first tank to the second tank. A fish is picked from the second tank at random. What is the probability that the transferred fish was orange given that the second fish we picked was orange?*

2 Random Variables

A **random variable** is a rule that assigns a number to each outcome of an experiment. These can be **finite discrete**:

infinite discrete:

continuous:

There are a number of things that we can describe easily (without calculus) for finite discrete random variables, and so we will work with this type.

The **probability distribution of a random variable** is a table of values for a random variable and their associated probabilities. A **histogram** is a graph of rectangles where the y-axis has probabilities and the x-axis represents values of the random variable. The areas of the rectangles give you the probability of an event.

Example 2.1 *A student takes a three question true false test and guesses randomly on all of the answers. Let X be the number of correct guesses.
Probability distribution of X :*

Histogram for X :

The **average** or **mean** of n numbers, $x_1, x_2, x_3, \dots, x_n$ is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}.$$

Example 2.2 *What is the mean of 2,3,6,2,4?*

Note: Your calculator can find this too: use LIST, MATH, 3:mean.

The **expected value** of X is found in the following way. Let X be a random variable that takes on values $x_1, x_2, x_3, \dots, x_n$ with associated probabilities $p_1, p_2, p_3, \dots, p_n$. Then we write the expected value as:

$$E(X) = x_1p_1 + x_2p_2 + \dots + x_np_n.$$

Example 2.3 *You pay \$2 for a lottery ticket where you pick four different numbers from 1 to 9. At the end of the day, they announce the four winning numbers. If you get exactly three numbers correct you win \$10. If you get all four numbers correct, you win \$100. Let X be your net winnings. Write a probability distribution for X :*

What is $E(X)$?

Often times people will discuss probabilities using the term “odds”. If $P(E)$ is the probability of an event occurring, then:

The odds of E occurring are $\frac{P(E)}{1-P(E)} = \frac{P(E)}{P(E^c)}$.

Sometimes this is said “odds in favor of E are $P(E)$ to $P(E^c)$ ”.

The odds against E occurring are $\frac{1-P(E)}{P(E)} = \frac{P(E^c)}{P(E)}$.

Sometimes this is said “odds against E are $P(E^c)$ to $P(E)$ ”.

Example 2.4 *The probability of candidate A winning an election is .35. What are the odds that candidate A wins?*

Alternately, if the odds in favor of E are a to b , then we can find the probability of E :

$$\frac{P(E)}{1-P(E)} = \frac{a}{b}$$

Example 2.5 *In the song “Secret Agent Man” we are told “Odds are he won’t live to see tomorrow”. If the odds that he won’t live to see tomorrow are 3 to 7, what is the probability that he’ll live until tomorrow (Wednesday)?*