Math 141: Business Math I	**FALL 2006**
Section 520	**Blocker 158, Tuesday and Thursday 3:55-5:10**
Week 7: 6.2-6.4 (part 2)	

1 Permutations!

A **permutation** of a set of objects is a way to arrange the objects in a definite order.

Example 1.1 *A game show host will let you win a brand new bicycle if only you can determine the price. You're told that the price has three digits: 3, 6, and 8. You need to determine the order of the digits.*
What are the different possibilities? How many are there?

In general, whenever we have n different things, we have n(n-1)(n-2)...1 ways of arranging them in order. This is so important, we've given it a name and symbol:

n factorial is written $n! = n(n-1)(n-2)...1$
When n=0, this is defined to be $0! = 1$.

On your calculator, go to MATH. Then go to PRB and pick selection 4, !. We don't always want to make a list of all n things, though. Sometimes we want to start with n items and pick (in order) r of them.

Example 1.2 *A magician is picking volunteers from the 12 person audience to help him with three different tricks (pick a card, fake sword through the head, levitating ball). He doesn't want to use the same volunteer twice, and each trick requires exactly one volunteer. How many ways can he pick these volunteers?*

This generalizes. The number of permutations of n distinct objects taken r at a time is

$$P(n,r) = \frac{n!}{(n-r)!}$$

On your calculator, go to MATH. Then go to PRB and pick selection 2, nPr. For example, to calculate $P(5,2)$ on your calculator, do 5 nPr 2.

2 Permutations of objects that aren't all distinct

We also can have situations where we permute objects that aren't distinct. For example, how many ways can we arrange the letters in the word COOL? Let's write them out:

This leads to a general formula for how we can arrange a collection of n objects where n_1 are identically of type 1, n_2 are of identically type 2,...n_r are identically of type r so that $n_1 + n_2 + ...n_r = n$. Then the number of distinct permutations of the entire set is

$$\frac{n!}{n_1!n_2!...n_r!}.$$

There's no button for this on the calculator, you need to use the ! one.

Example 2.1 *On Thursday, Habitat for Humanity had 12 people who volunteered to help build a house. They needed seven people to paint, two to install baseboards, and three to clean the yard area. How many ways could they assign these jobs?*

3 Combinations

There are also times when we pick a set of distinct objects from a larger collection without caring about the order. The number of combinations of r objects taken from a group of n distinct objects is:

$$C(n,r) = \frac{n!}{r!(n-r)!}.$$

On your calculator, go to MATH. Then go to PRB and pick selection 3, nCr.

Notice that this is the same as $\frac{P(n,r)}{r!}$; we take the number of ways to pick r things in order, and then divide by the number of possible orders of them.

Example 3.1 *A magician is picking three volunteers from the 12 person audience to help him with one trick ("saw a lady in half"). Each volunteer will do exactly the same thing. How many ways can he pick these volunteers?*

Example 3.2 *Last year I had a giant bowl of candy for tricker treaters. It had 22 Snickers, 35 Kit Kats, and 20 Almond Joy bars in it. How many ways can you take 5 candy bars from this bowl?*

How many ways can you take 3 Kit-Kats, 1 Almond Joy, and 1 Snickers?

How many ways can you take 5 candy bars so that all five have nuts? (Almond Joy and Snickers both have nuts)

How many ways can you take 5 of the same kind of candy bars?

4 Figuring out which to use

Example 4.1 *A bookshelf has eight different books on it. How many ways can one kid take a book from it?*

How many ways can one kid take three books from it?

Example 4.2 *A bookshelf has eight different books on it. How many ways can eight kids each take a book from it?*

How many ways can three kids go up and each take a book from it?

Example 4.3 *A bookshelf has five copies of "Go, Dog, Go", two copies of "The Little Engine that Could", and one copy of "The Pokey Little Puppy". How many ways can eight kids each take a book from it?*

How many ways can one kid take a book?

A standard deck of cards has 52 cards in four suits (club, diamond, heart, spades) and thirteen values (2,3,4,5,6,7,8,9,10,jack, queen, king, ace).

Example 4.4 *How many ways can we form a 5 card poker hand?*

How many ways can we form a 5 card hand where four of the cards are aces?

How many ways can we form a 5 card hand where each card has a different value?

How many ways can we form a 5 card hand where at least two cards have the same value?