Path planning and source detection in unknown environments

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Joint work with Martin Burger, Yana Landa, and Nick Tanushev

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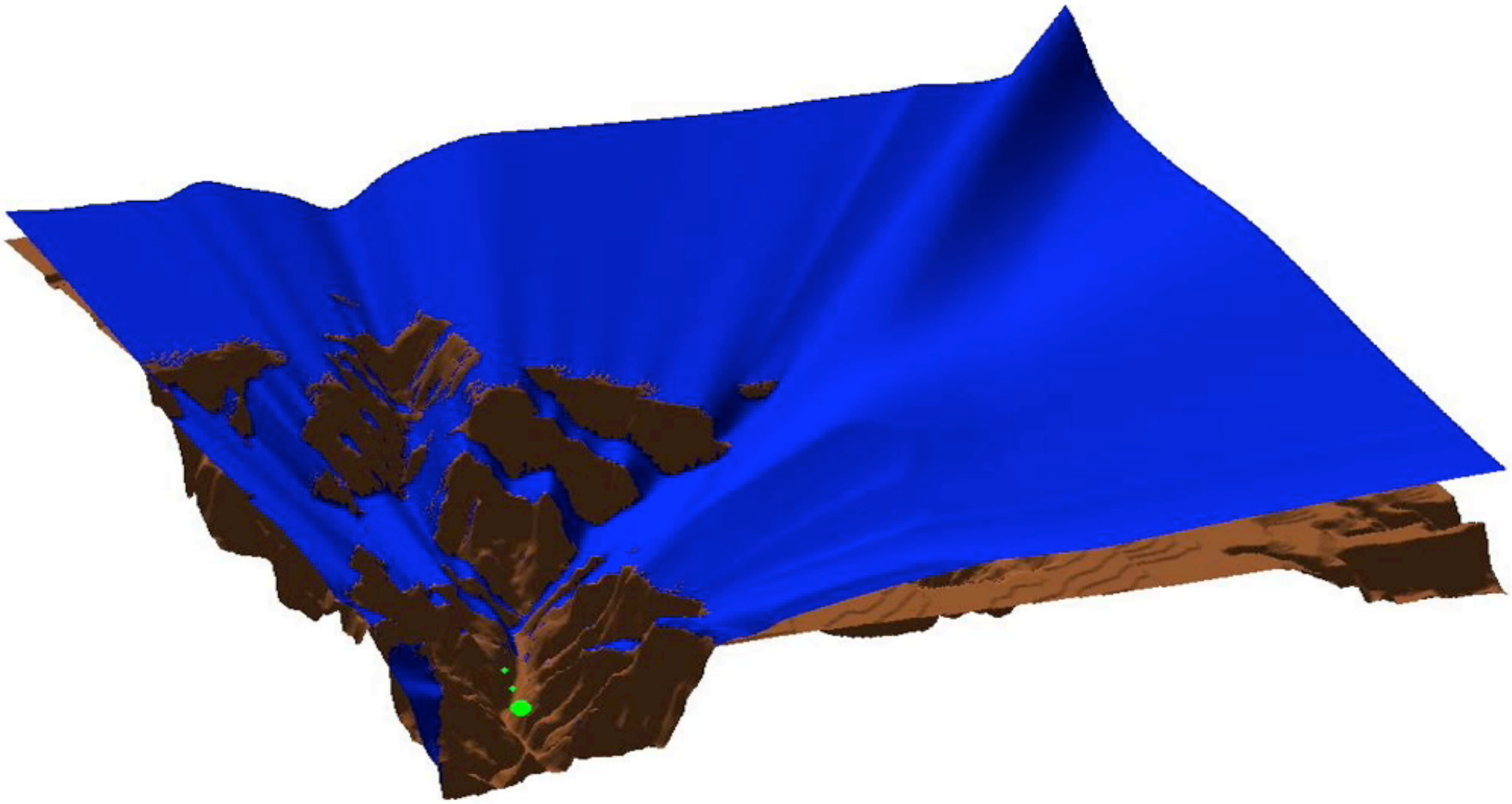
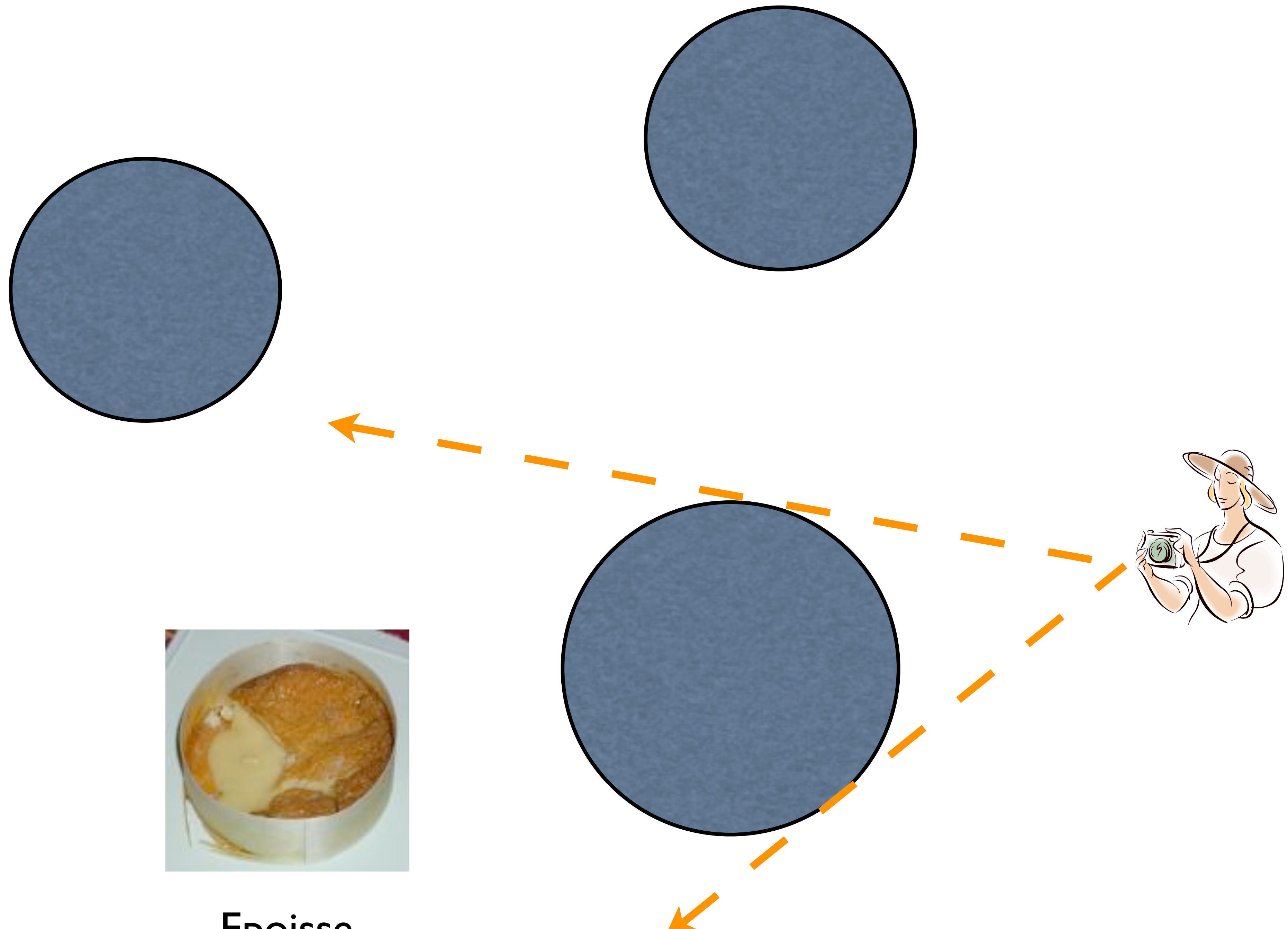
Exploration/surveillance



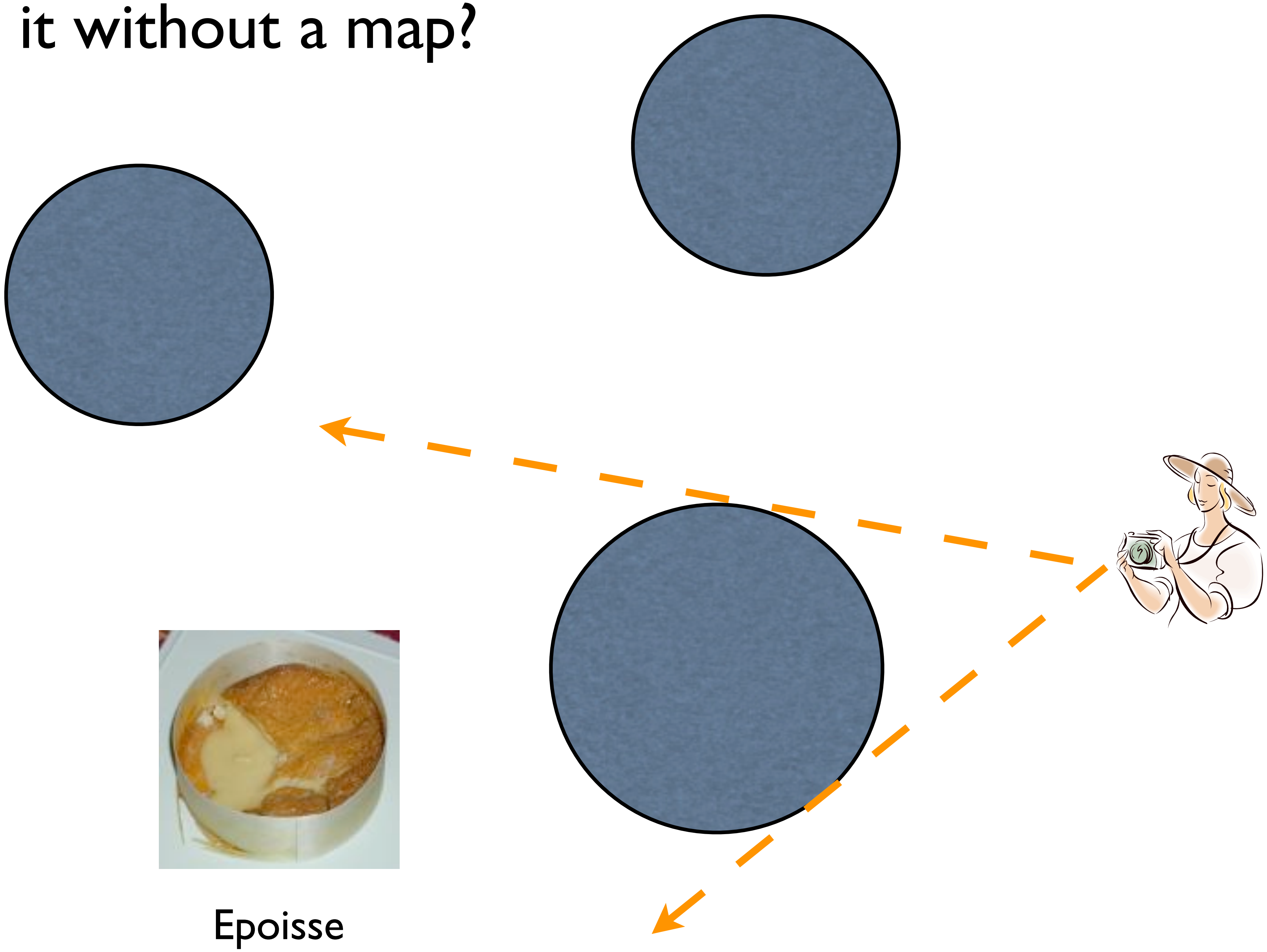
Shortest path to see a smelly object



Epoisse

Shortest path to see a smelly object

Do it without a map?



Epoisse

Major components

- Computing visibility (skipped in this talk) $\phi(x, O)$
  - when a map is available
  - without a map
- Path planning
  - map out and inspect the domain
  - discover and visually inspect the source(s)
  - do it “well” -- optimization, adaptivity

Defining visibility

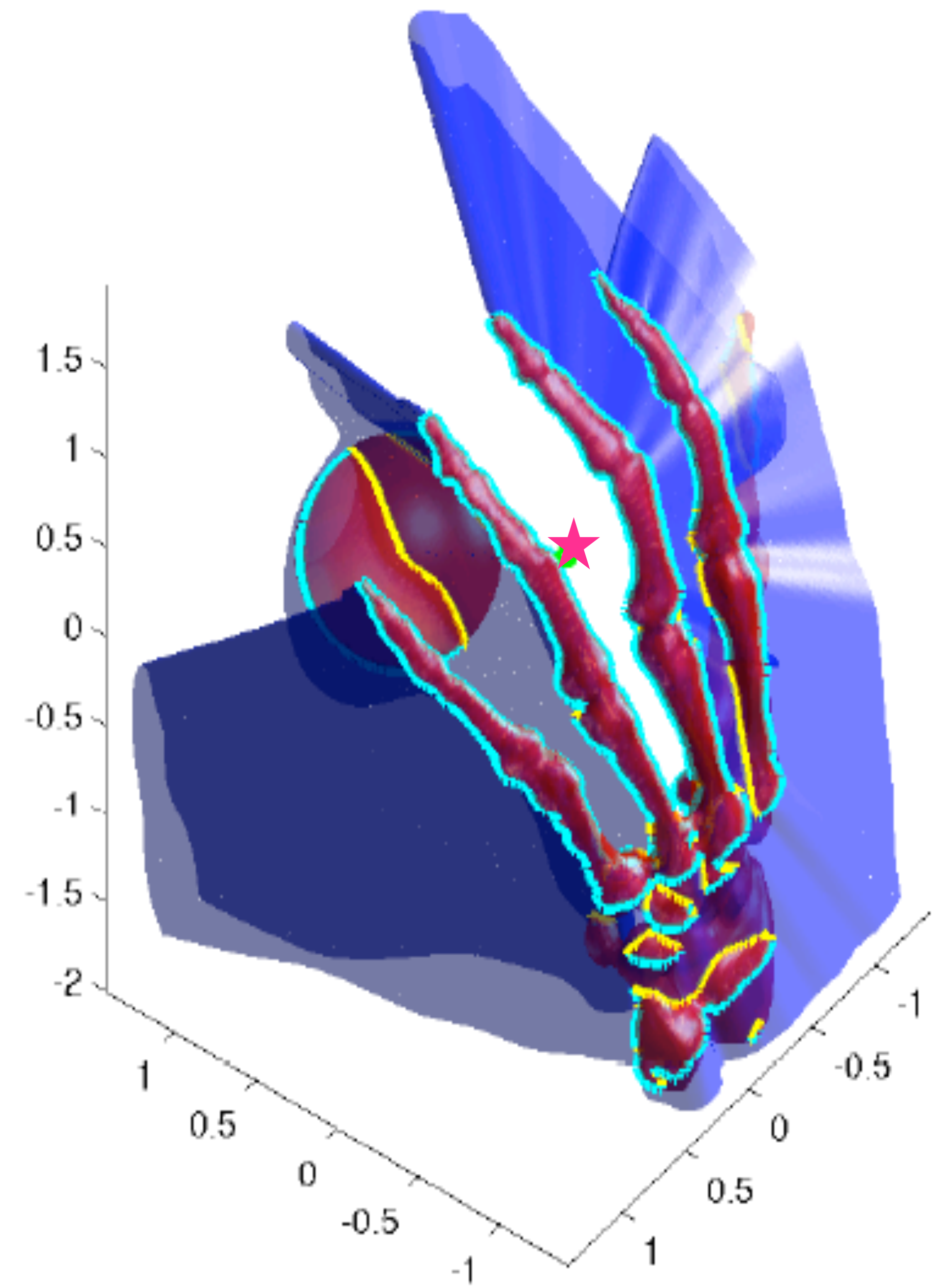
Obstacle: $\Gamma = \partial\Omega = \partial\{\Psi < 0\}$

Observing location: O

Construct a continuous function $\phi(y, O)$

$$\{y : \phi(y,O) < 0\} \;=\; \text{occlusion from O}$$

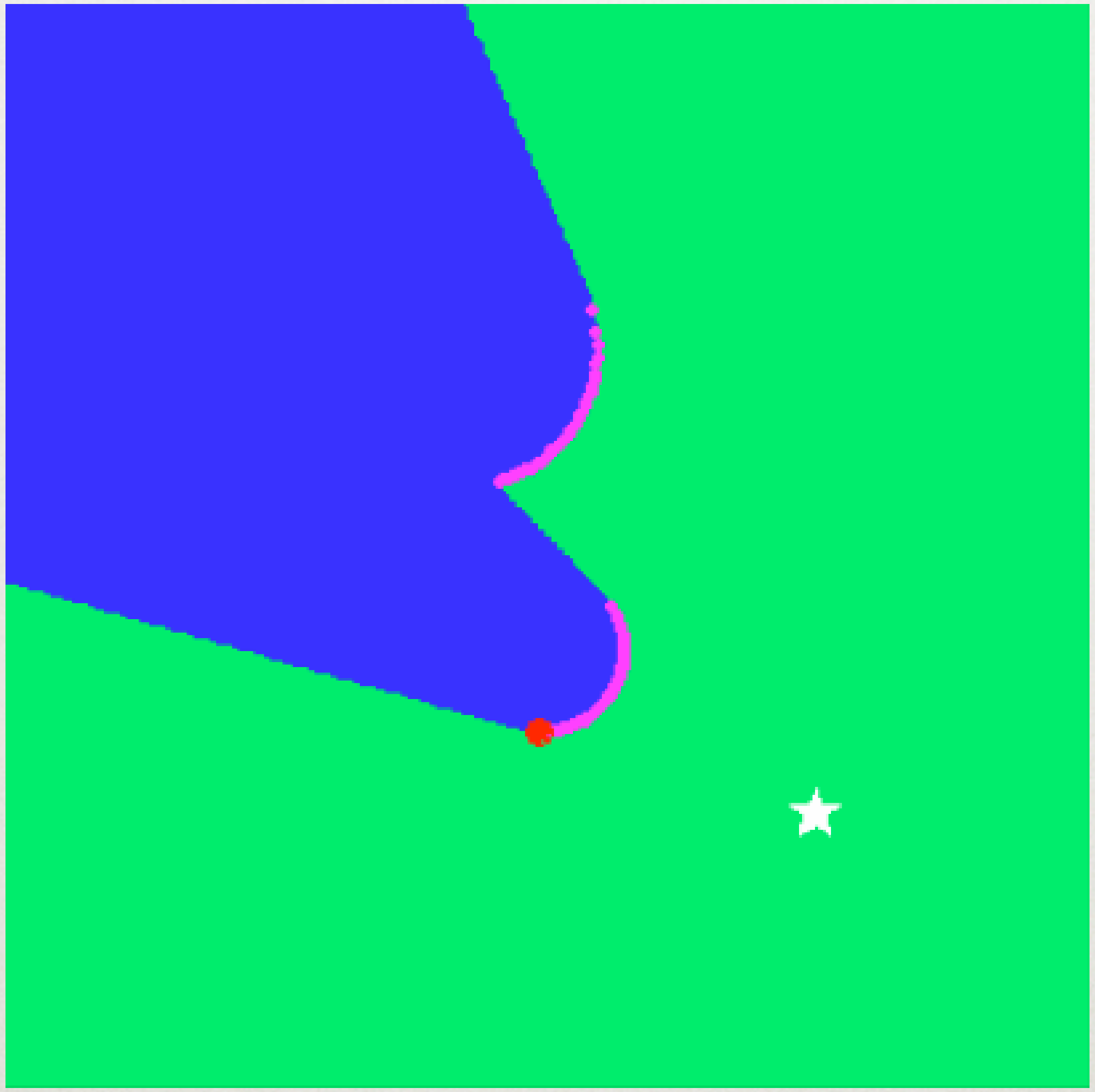
$$\phi(x,y) = \phi(y,x)$$ (reciprocity)

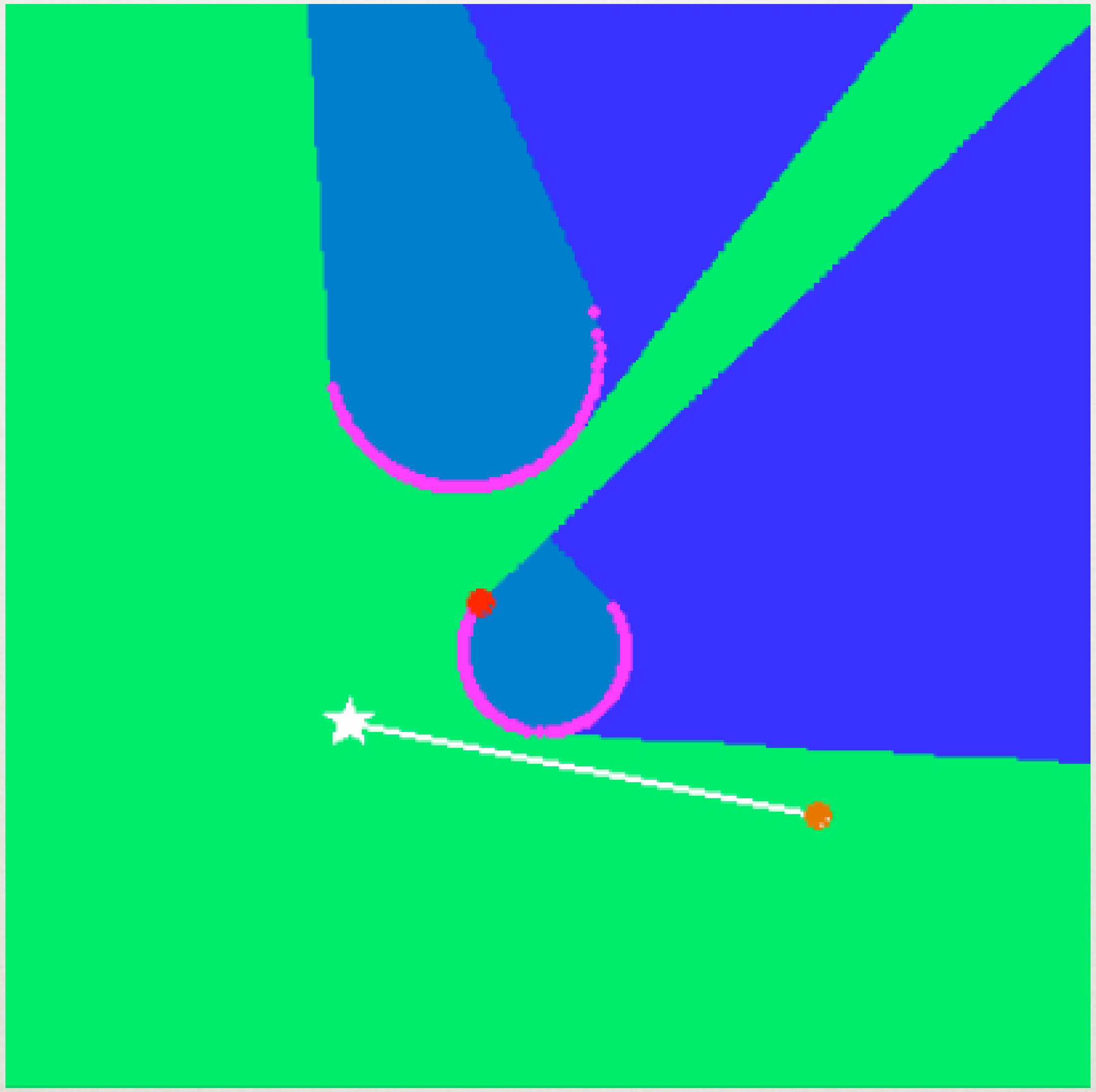


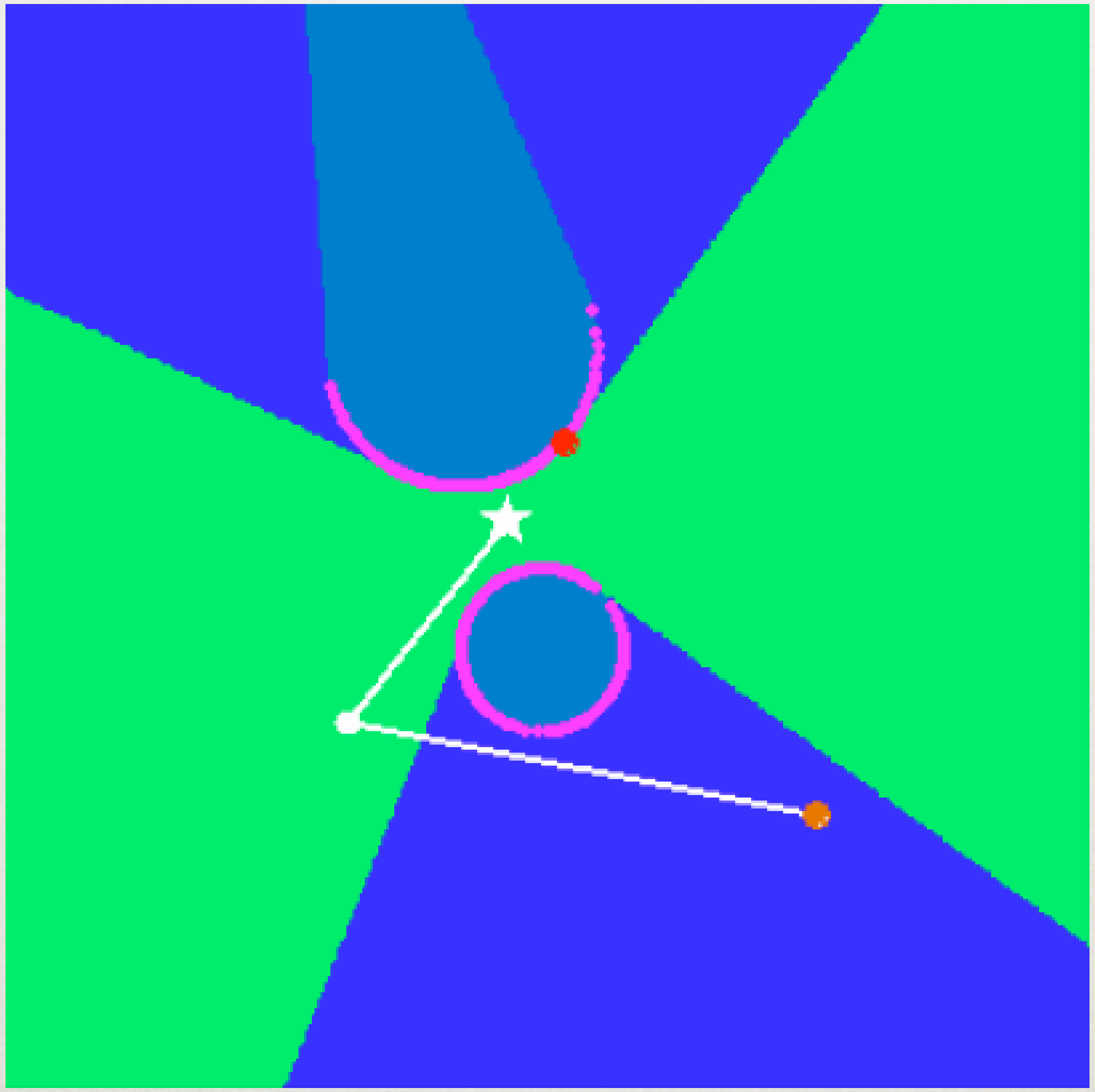
Related work

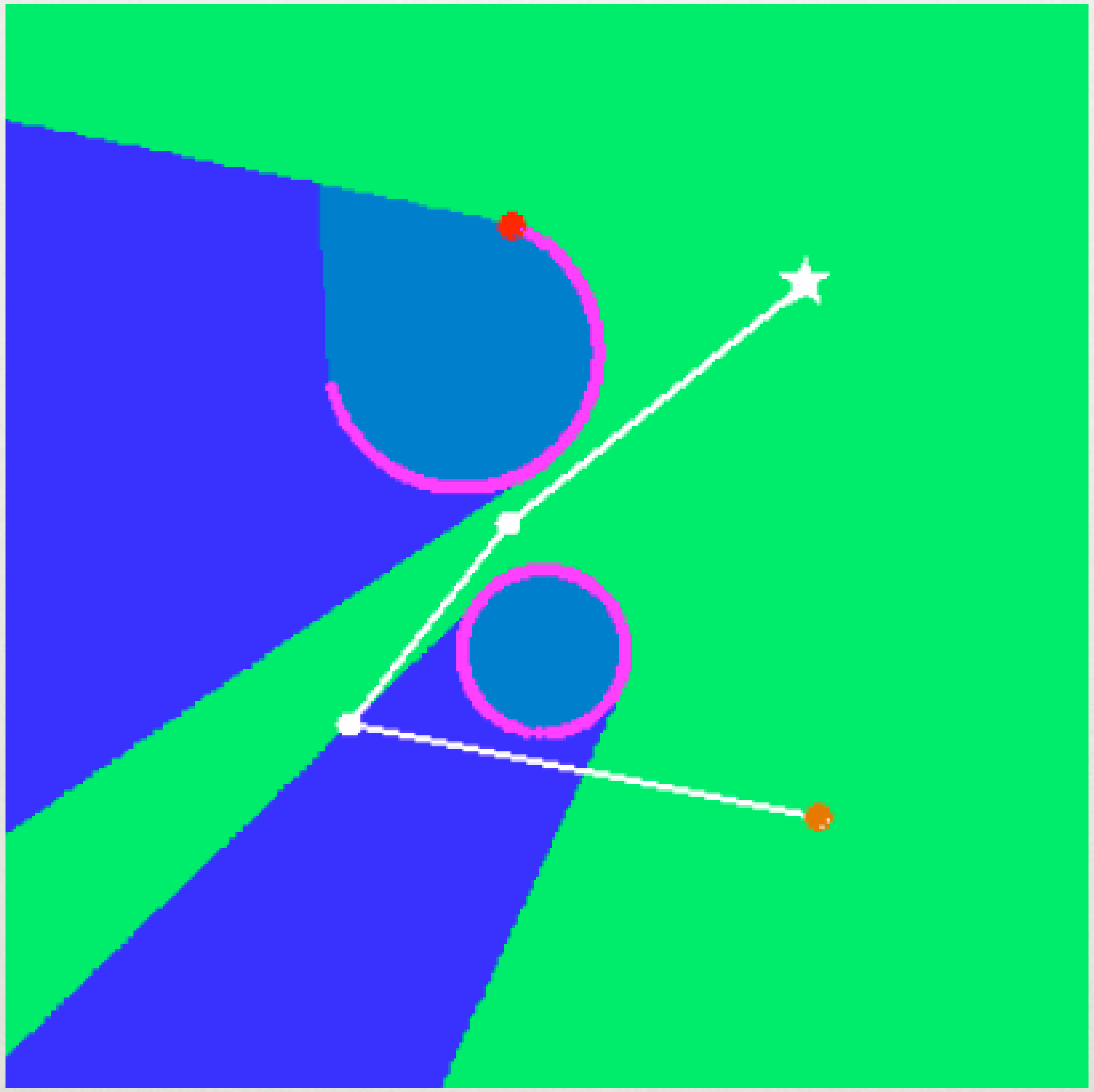
- Computing visibility:
- Algorithms for handling surfaces:
- Geometry: convexity and topology of the illuminated region of a solid surface [Ghomi]

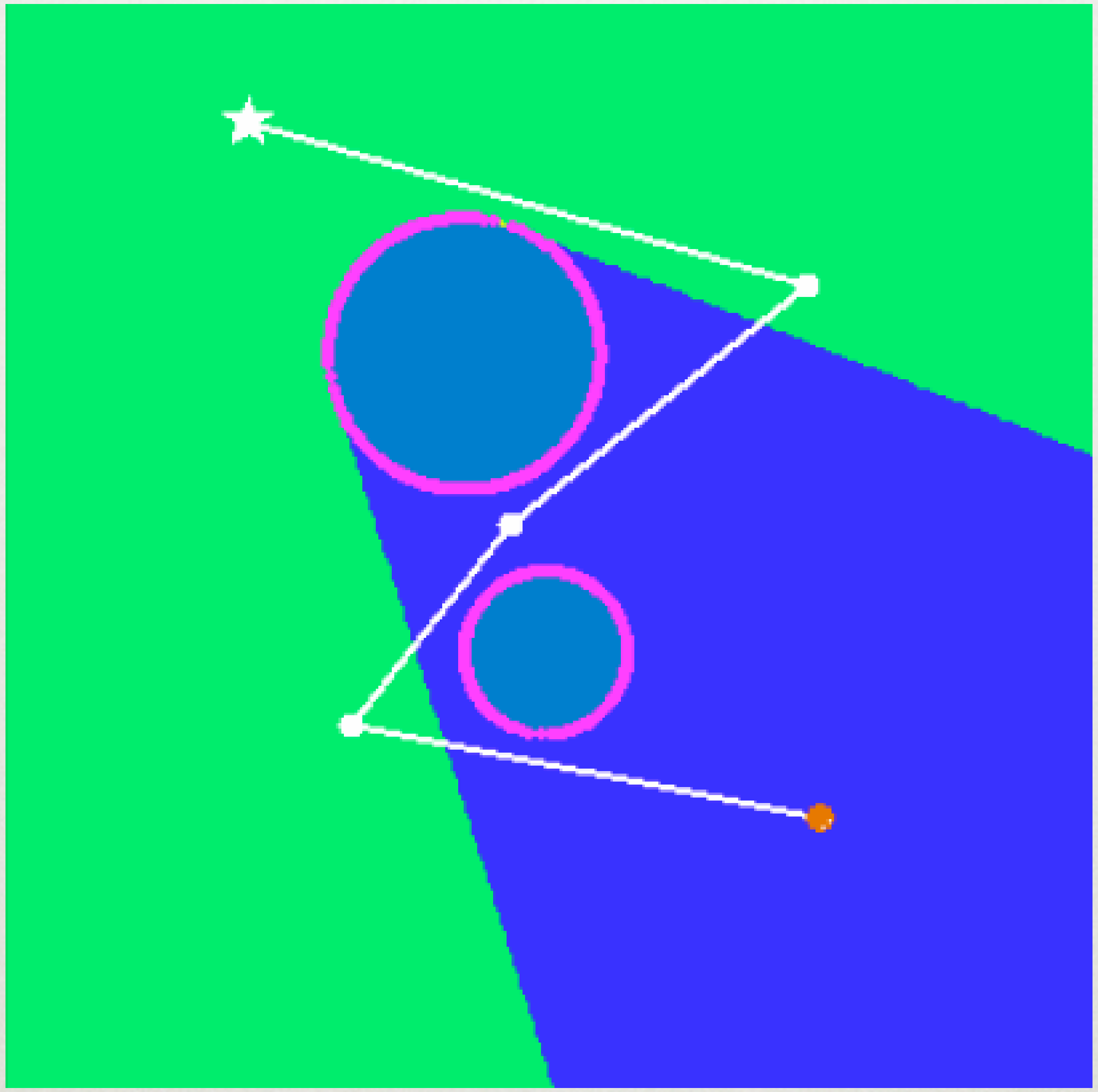
- Path planning:
  Hamilton-Jacobi, computational geometry, boundary, medial axes approaches. [LaValle][Landa-Tsai]
- Source detection:
  Inverse source problems
  [Yamamoto][Santosa-Symes][Isakov] and many others



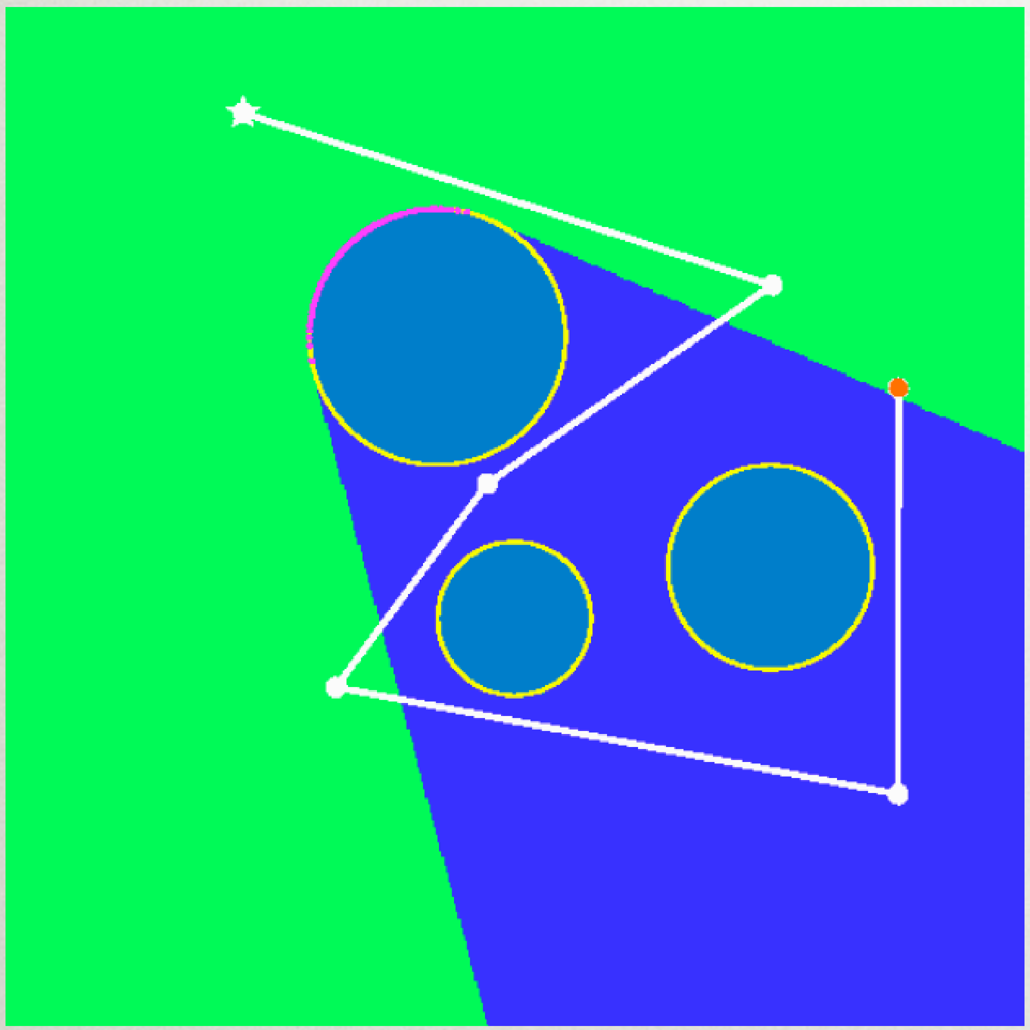


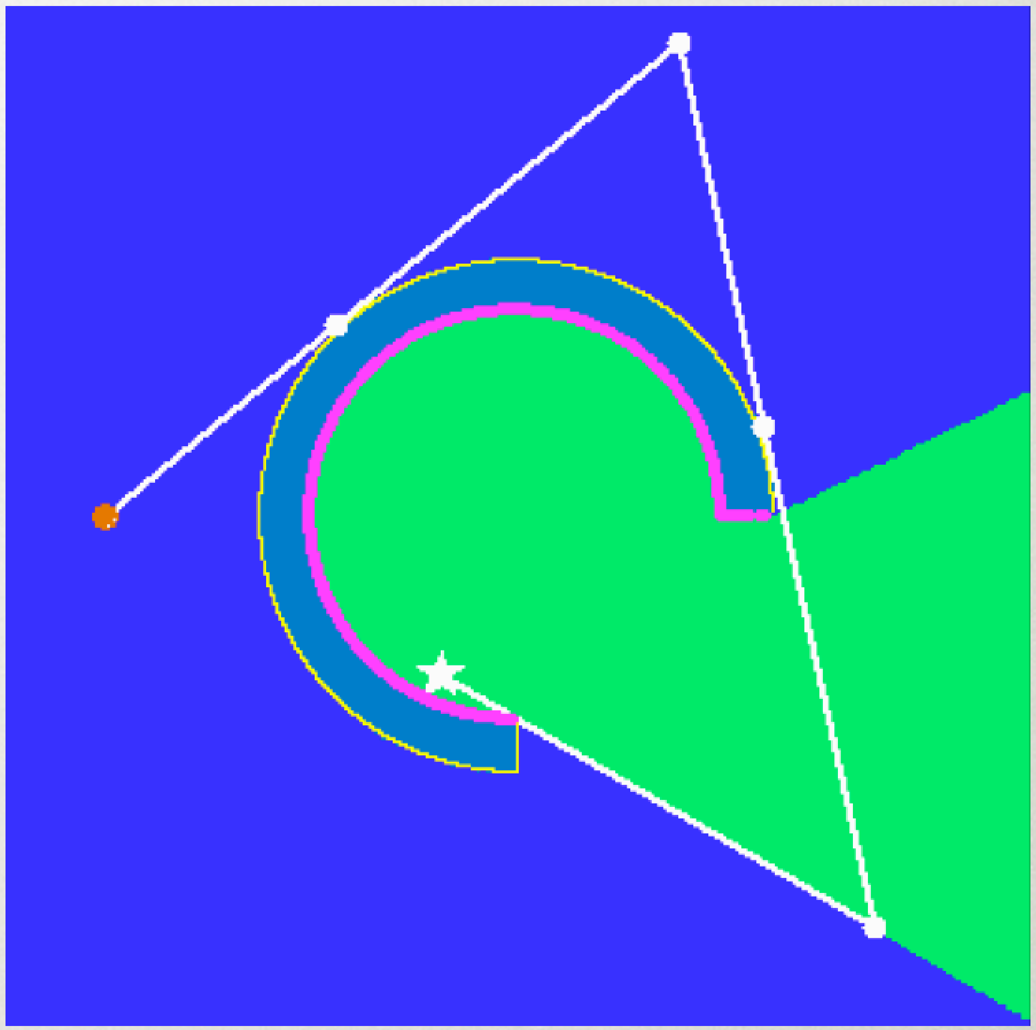


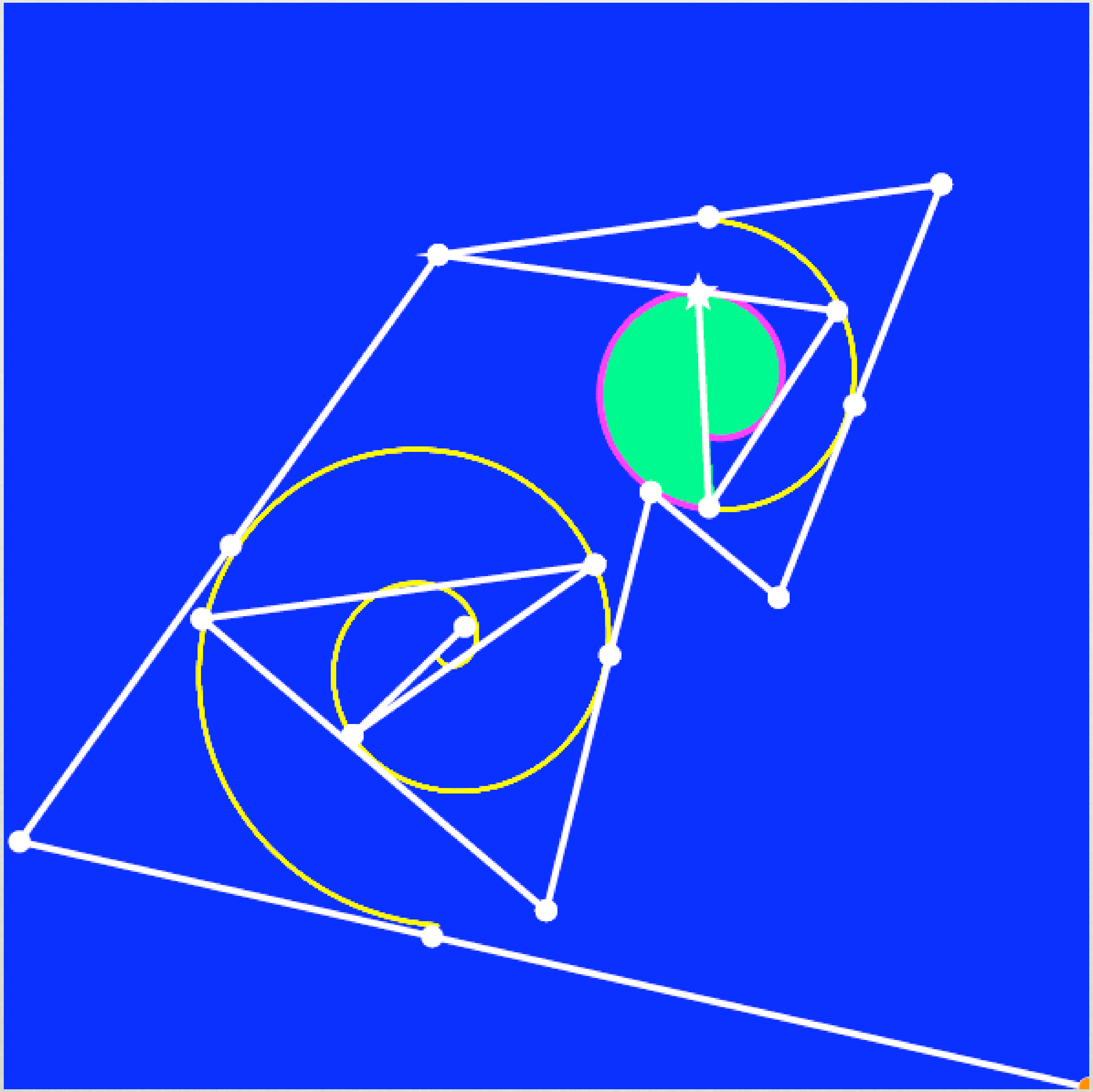




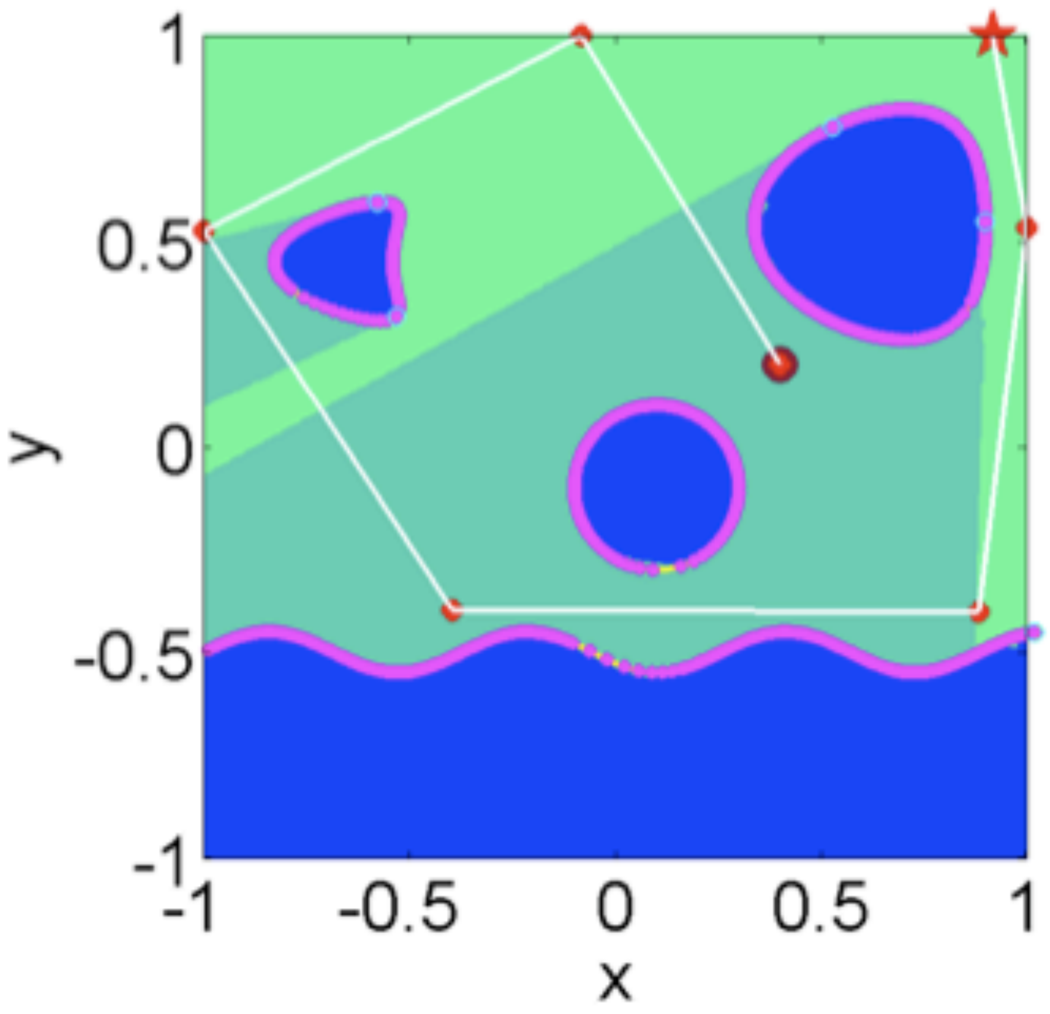
Obstacles reconstructed by an ENO technique.

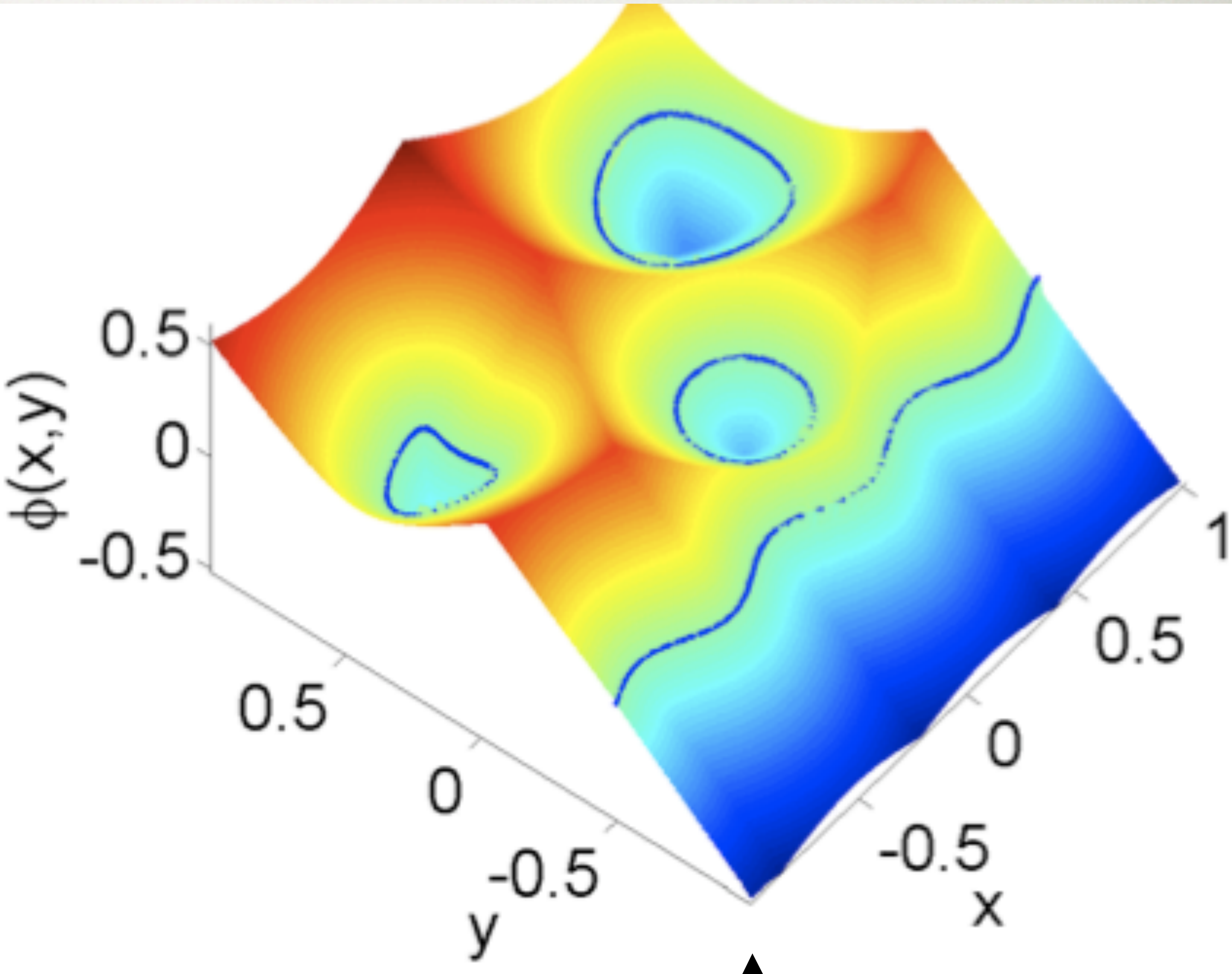






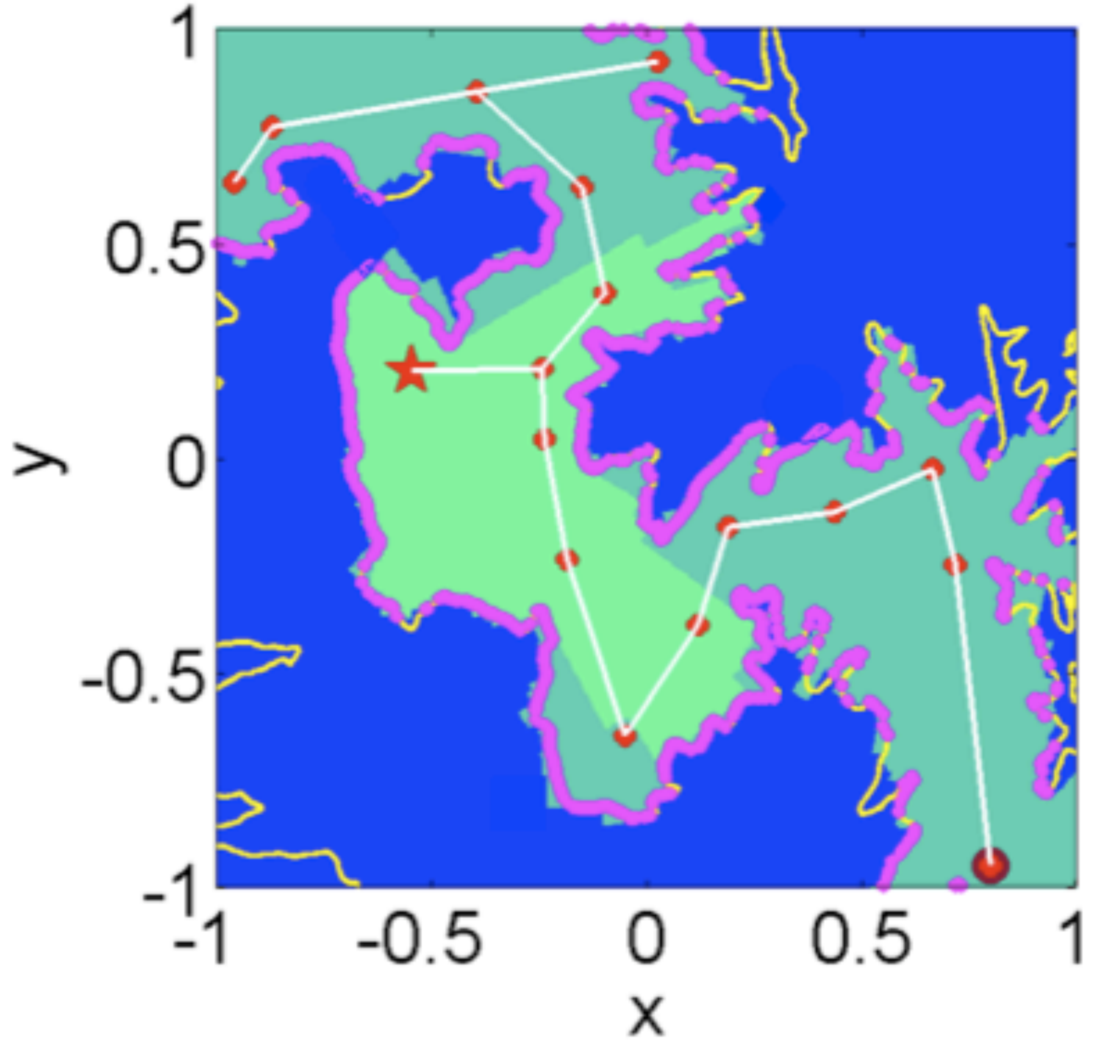
Creating a map

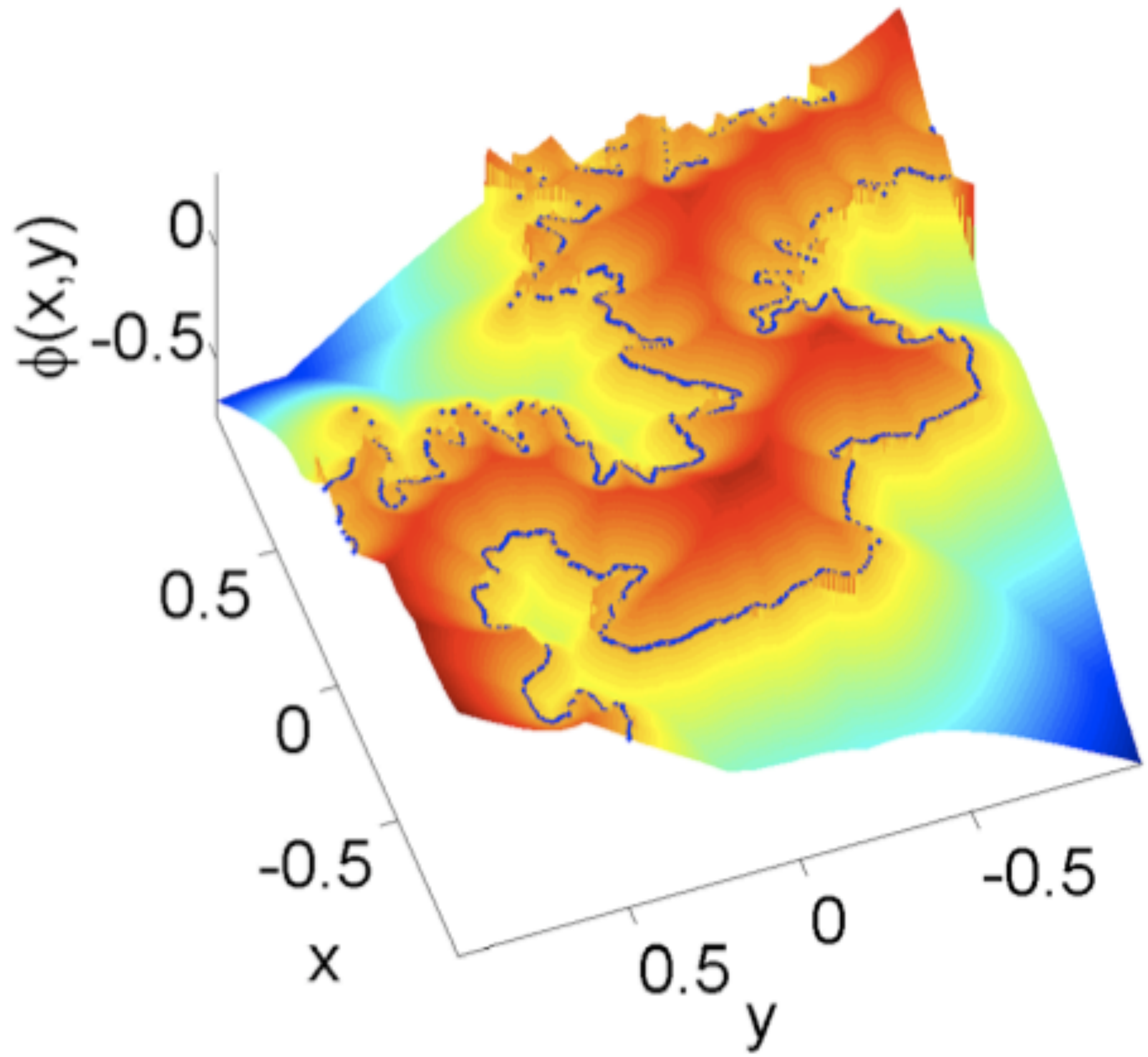




Level set representation of the environment.

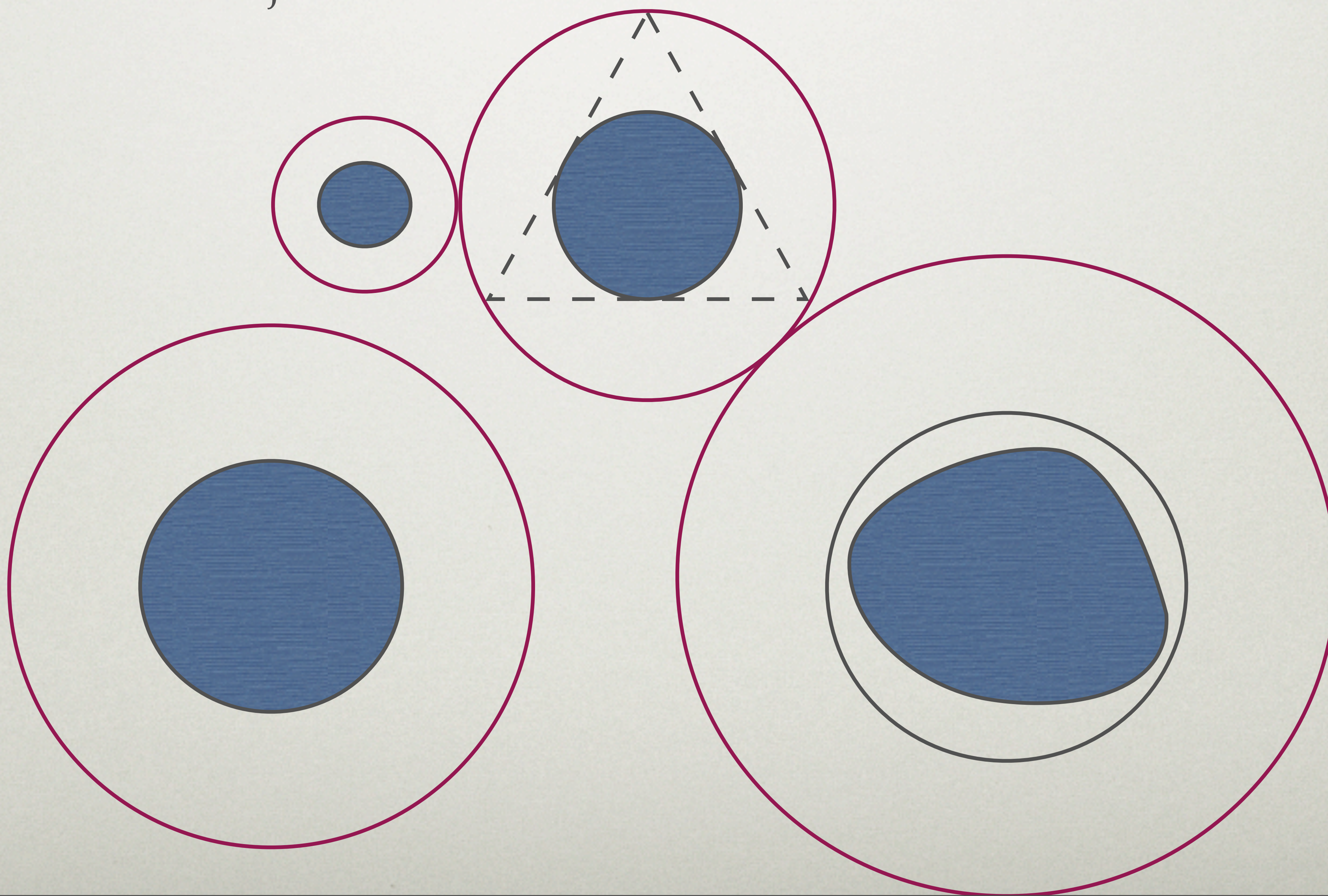
Simulation with real data





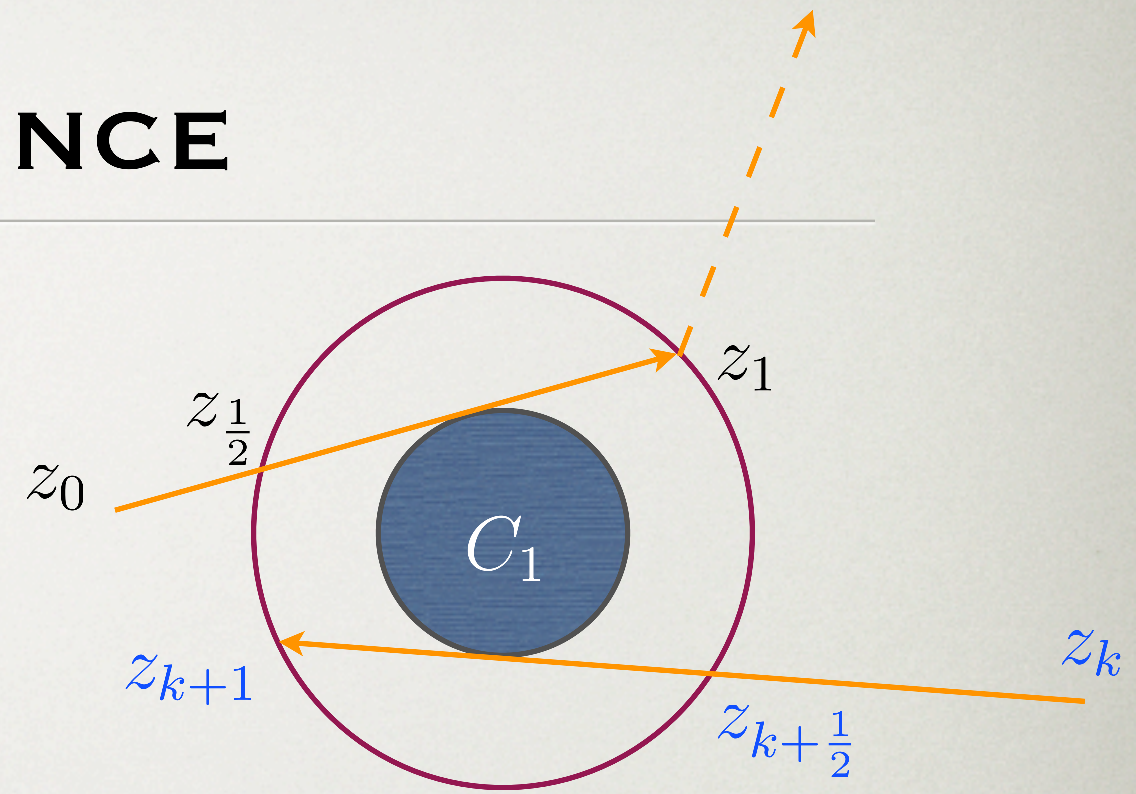
Convergence theory

- Single observer
- Convex objects
- Separation: non-overlapping disks, non-overlapping inscribing triangles
- Finite number of objects

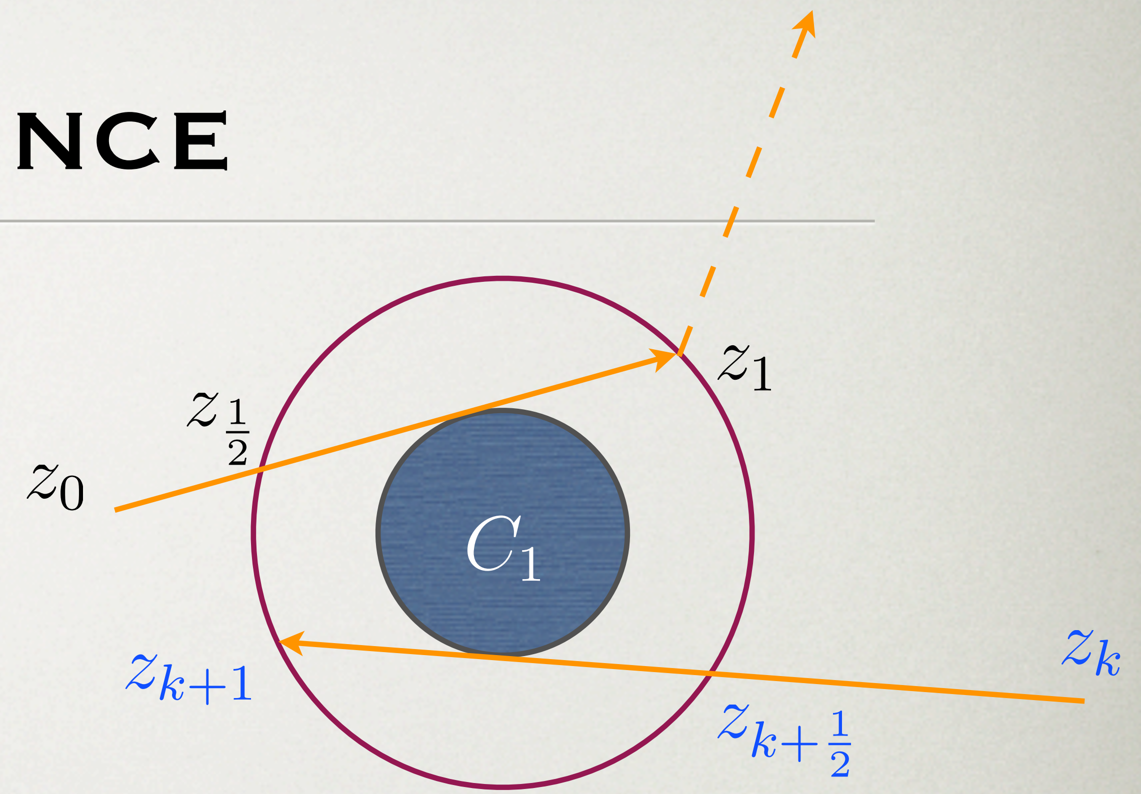


Convergence

Lemma: If for some $k > 1$, $z_{k+1} \in C'_1$,
then C_1 is entirely seen from $\{z_{\frac{1}{2}}, z_1, z_{k+\frac{1}{2}}, z_k\}$

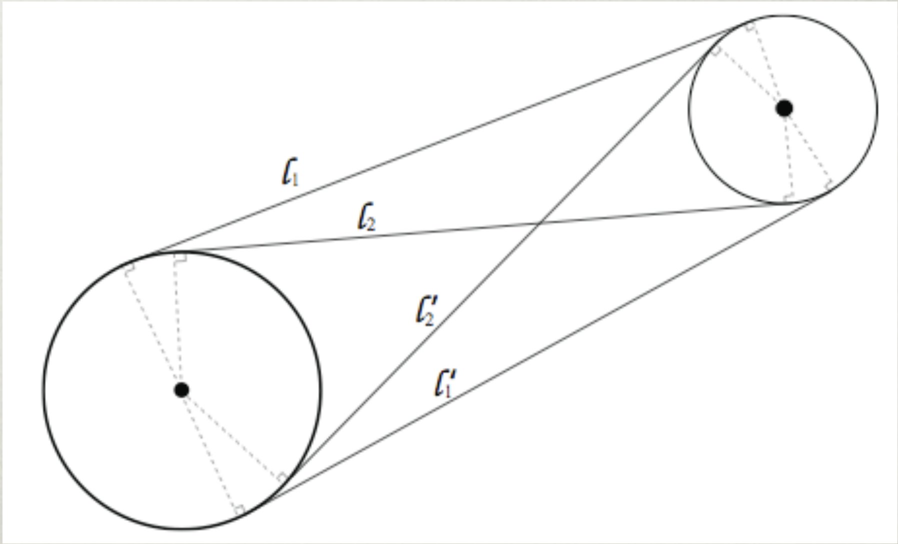


Convergence

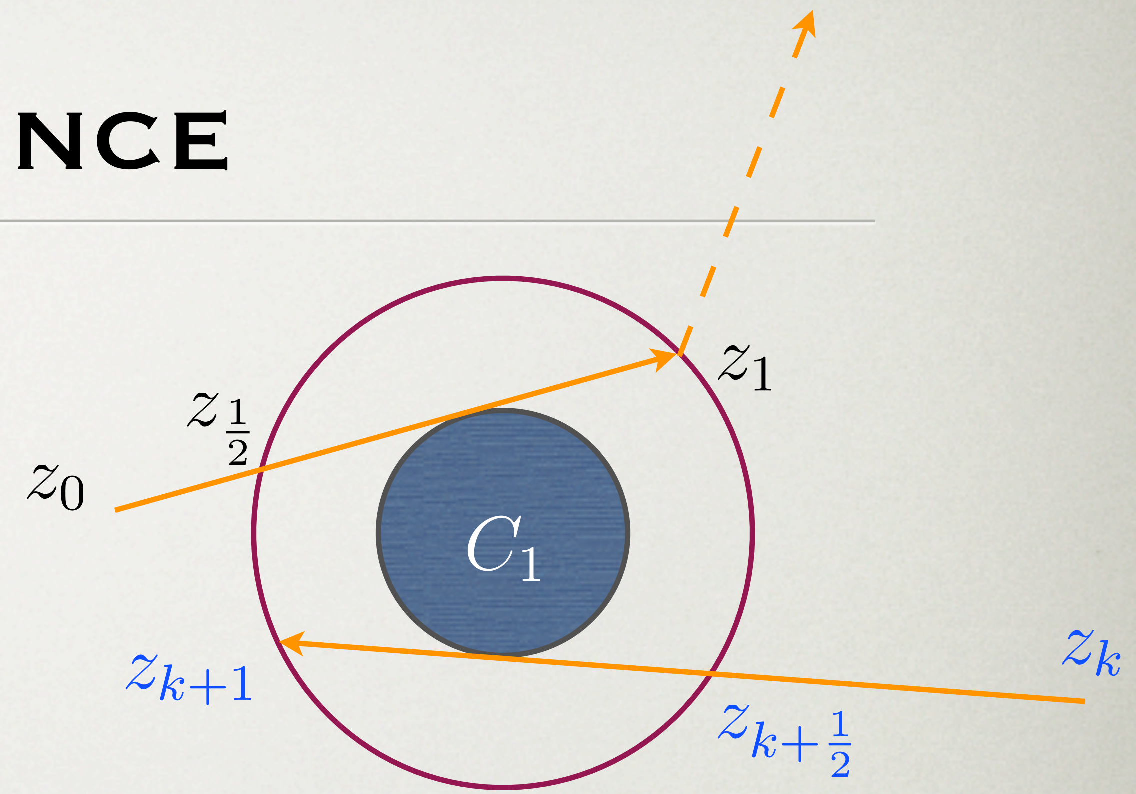


Lemma: If for some $k > 1$, $z_{k+1} \in C'_1$, then C_1 is entirely seen from $\{z_{\frac{1}{2}}, z_1, z_{k+\frac{1}{2}}, z_k\}$

Lemma: In the process of exploring C_j, the observer must detect at least one edge point on every neighbor of C_j.



Convergence

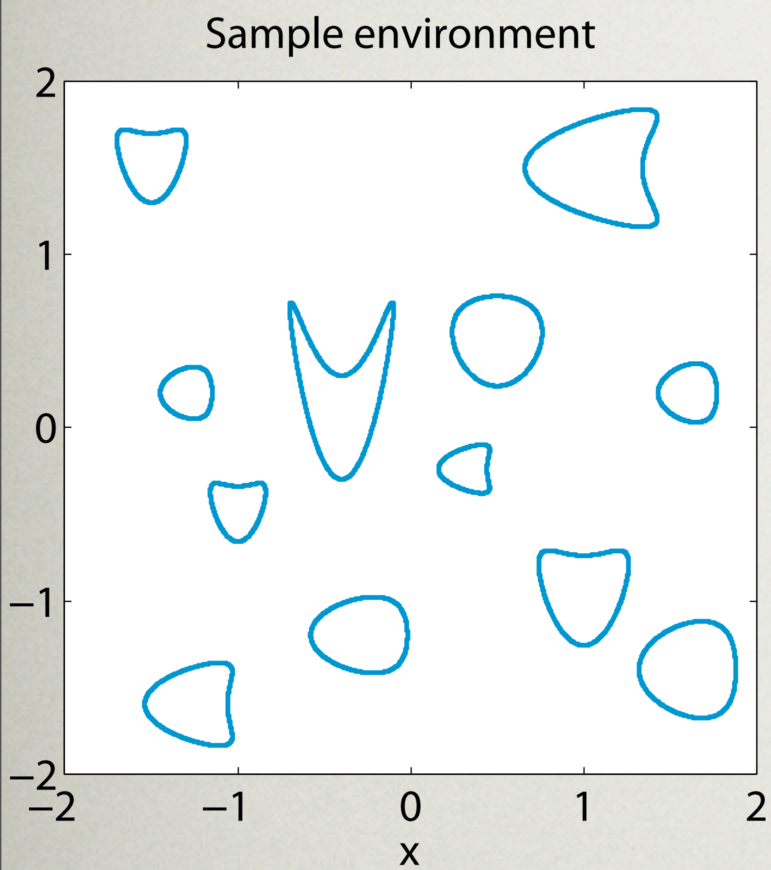


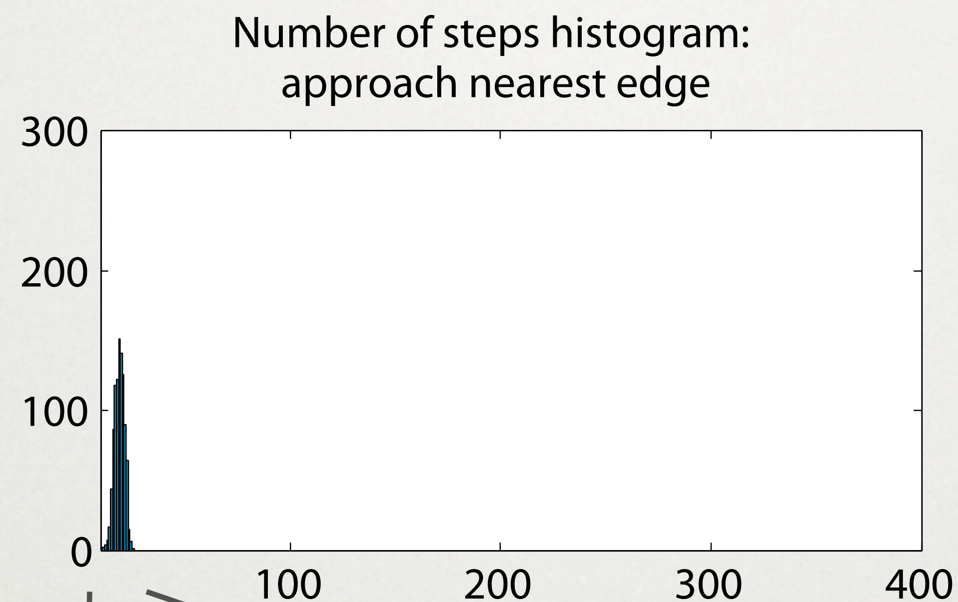
Lemma: If for some $k > 1$, $z_{k+1} \in C_1'$, then C_1 is entirely seen from $\{z_{\frac{1}{2}}, z_1, z_{k+\frac{1}{2}}, z_k\}$

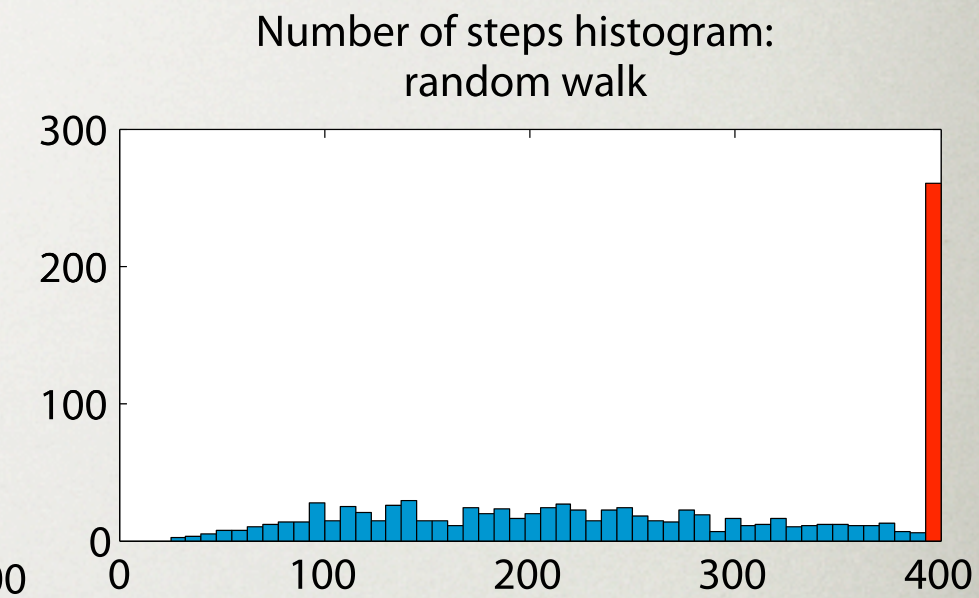
Lemma: Every disk will have at least one edge point labeled on it before algorithm terminates

Proposition:
The entire environment B_R is explored at the termination of the algorithm
$\Longleftrightarrow$
The observer has seen the boundary of every obstacle at the termination.

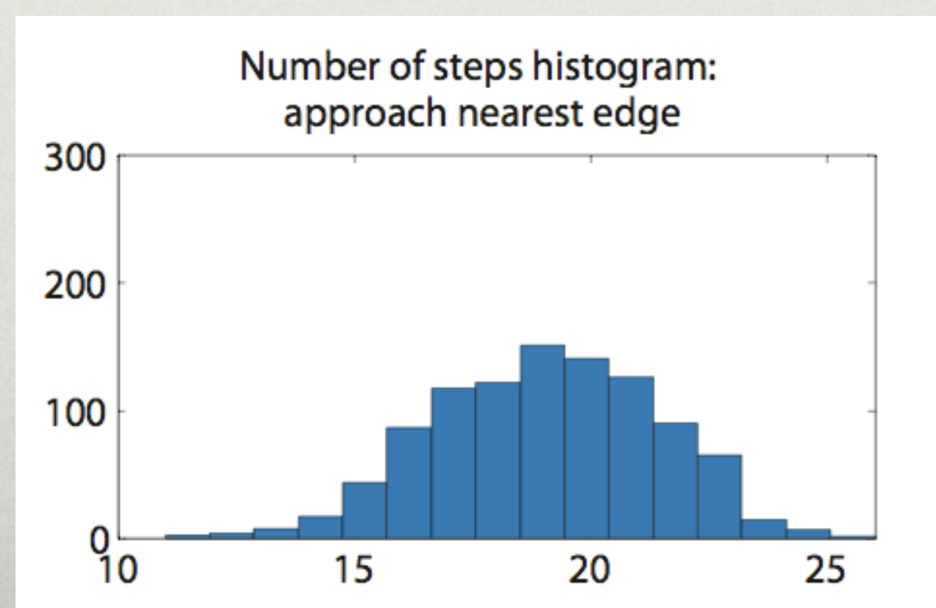
Some statistics



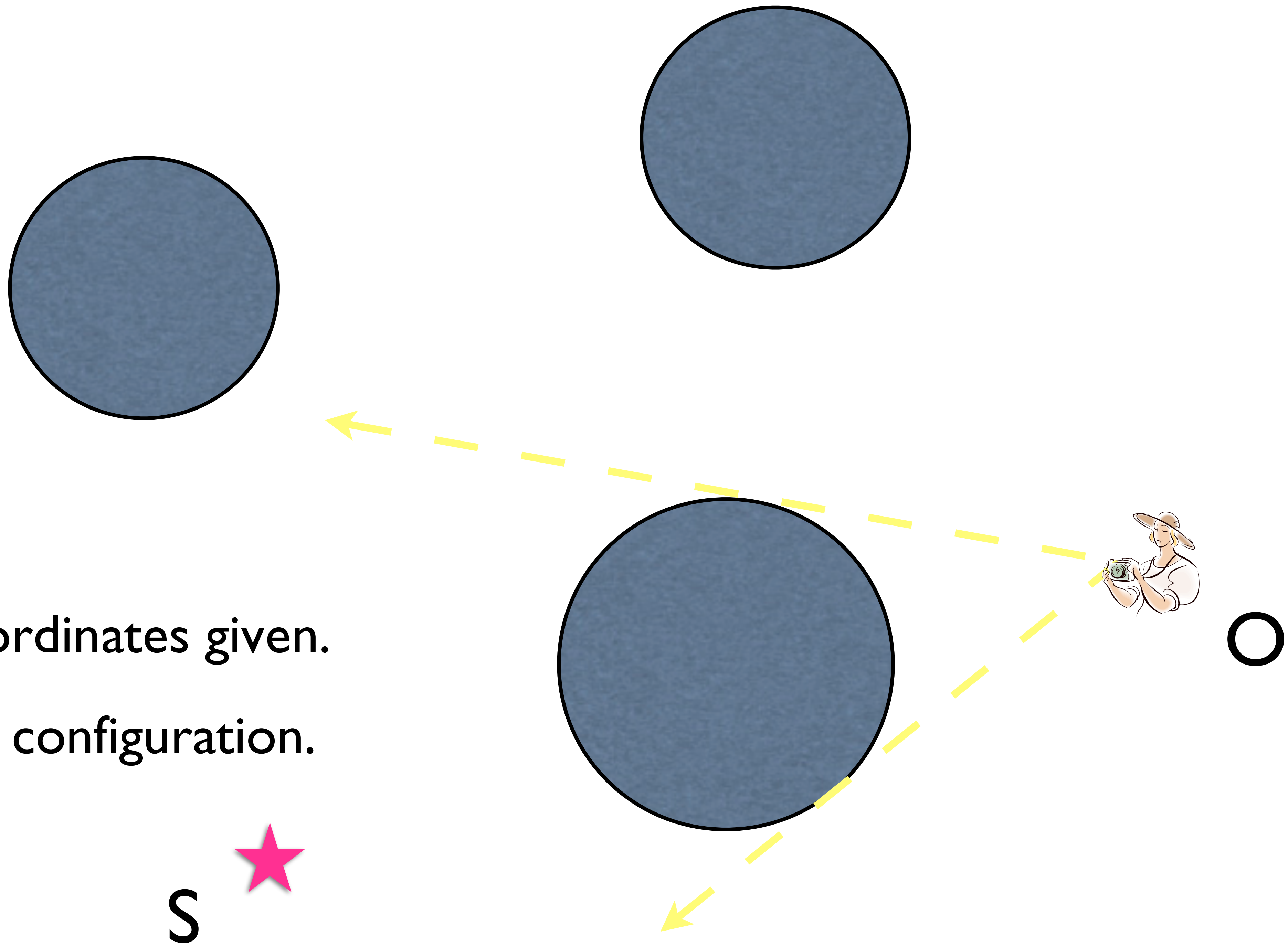




1000 independent runs



Shortest path to see an object



Target's GPS coordinates given.

Known obstacle configuration.

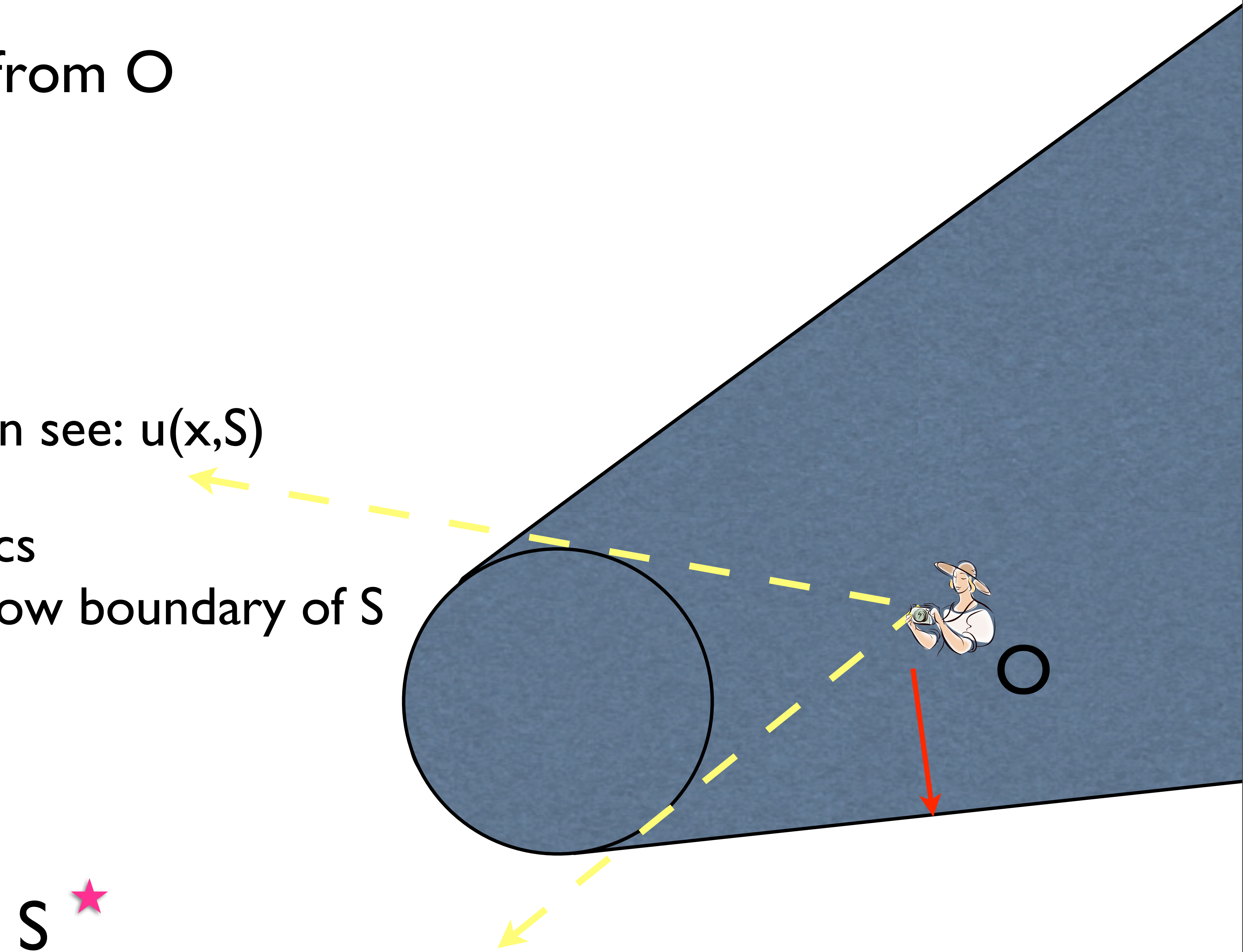
Shortest path to see an object

u(x,O): visibility from O

reciprocity:
u(x,y)=u(y,x)

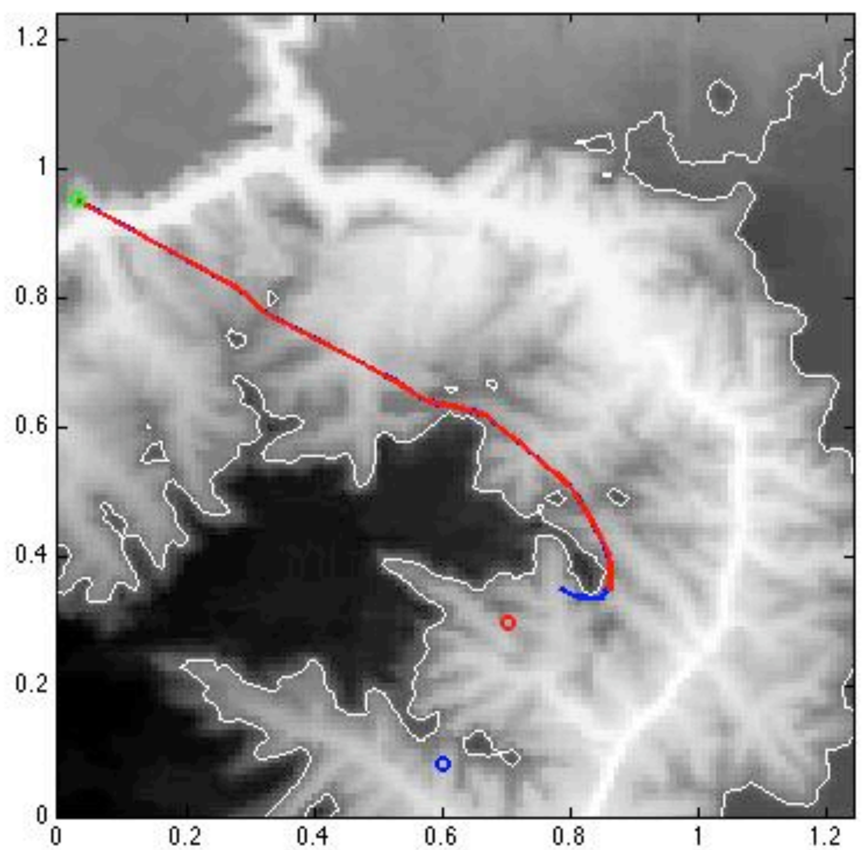
Compute what S can see: u(x,S)

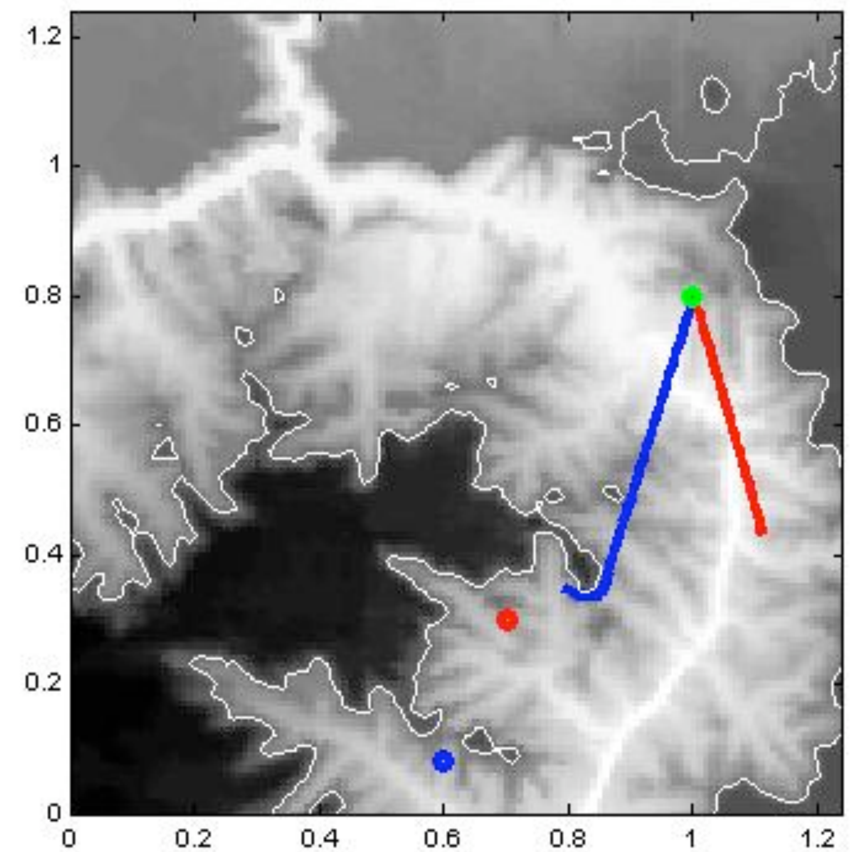
Compute a geodesics
from O to the shadow boundary of S



Shortest path to see multiple objects

- Extension to multiple observers
- Algorithm determines the existence of solutions.

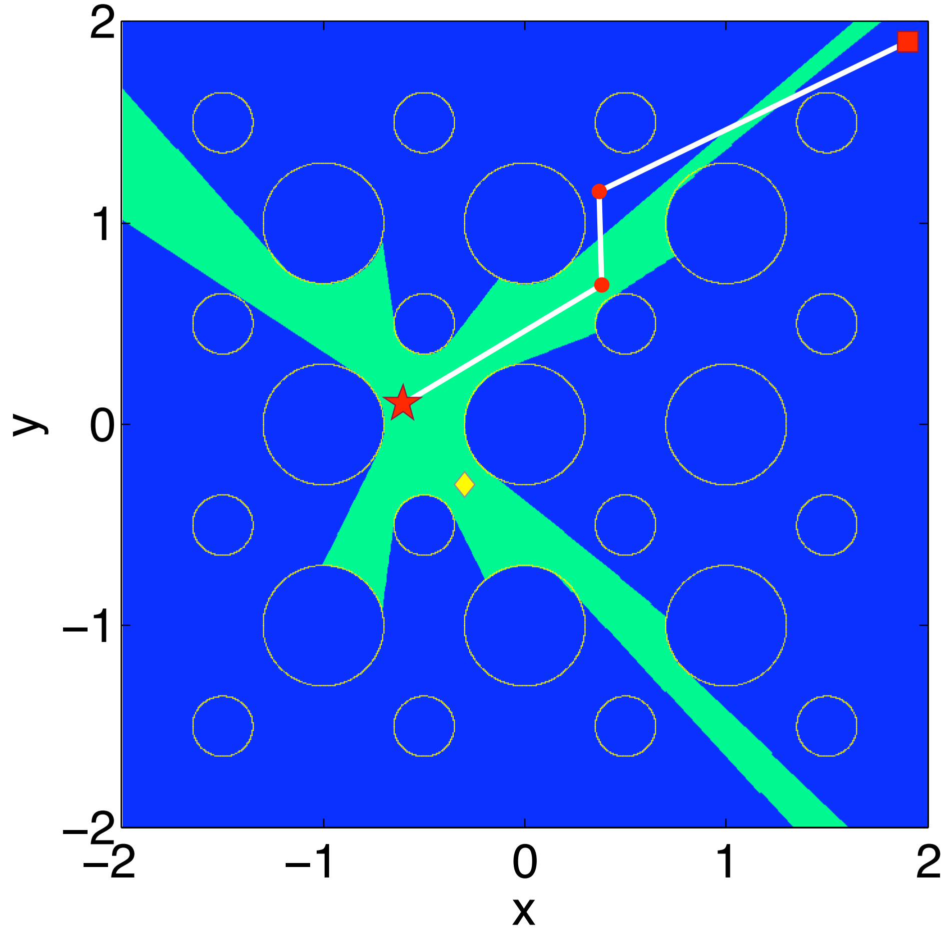




Target detection in unknown environments

- Obstacles in the domain are unknown
- Given the location of the target: S
- Visibility from observing location available

Move around to see the target (S) as soon as possible.



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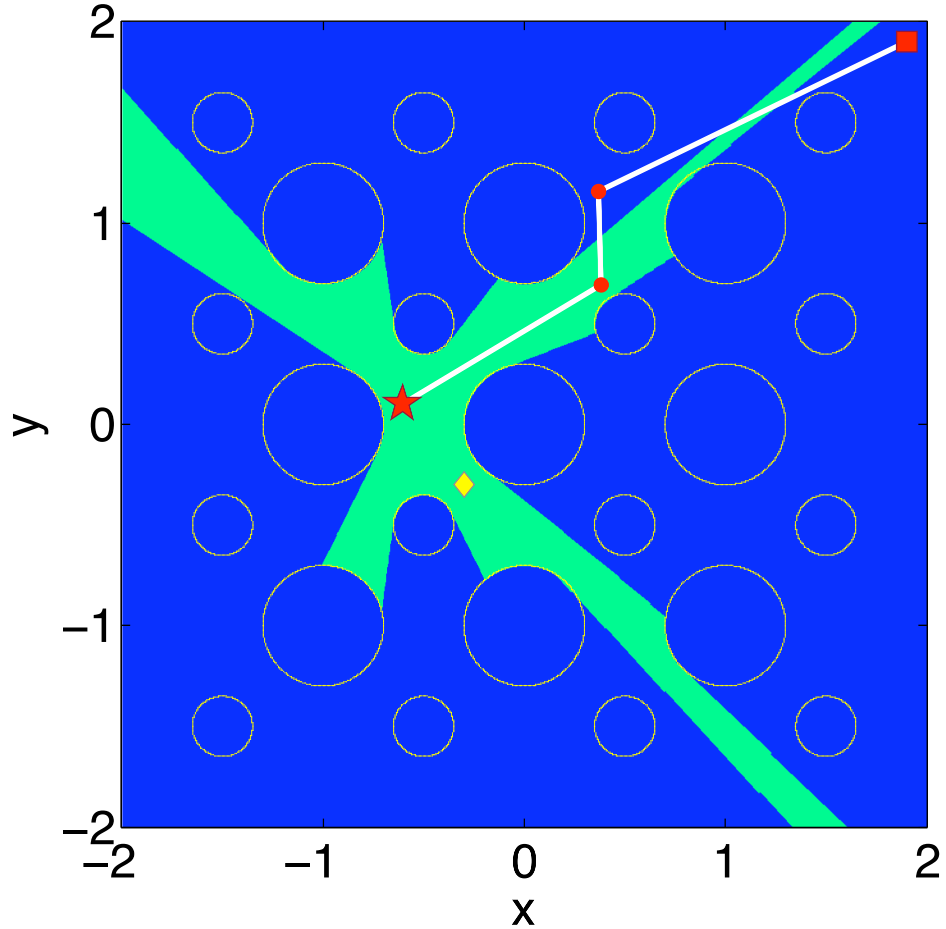
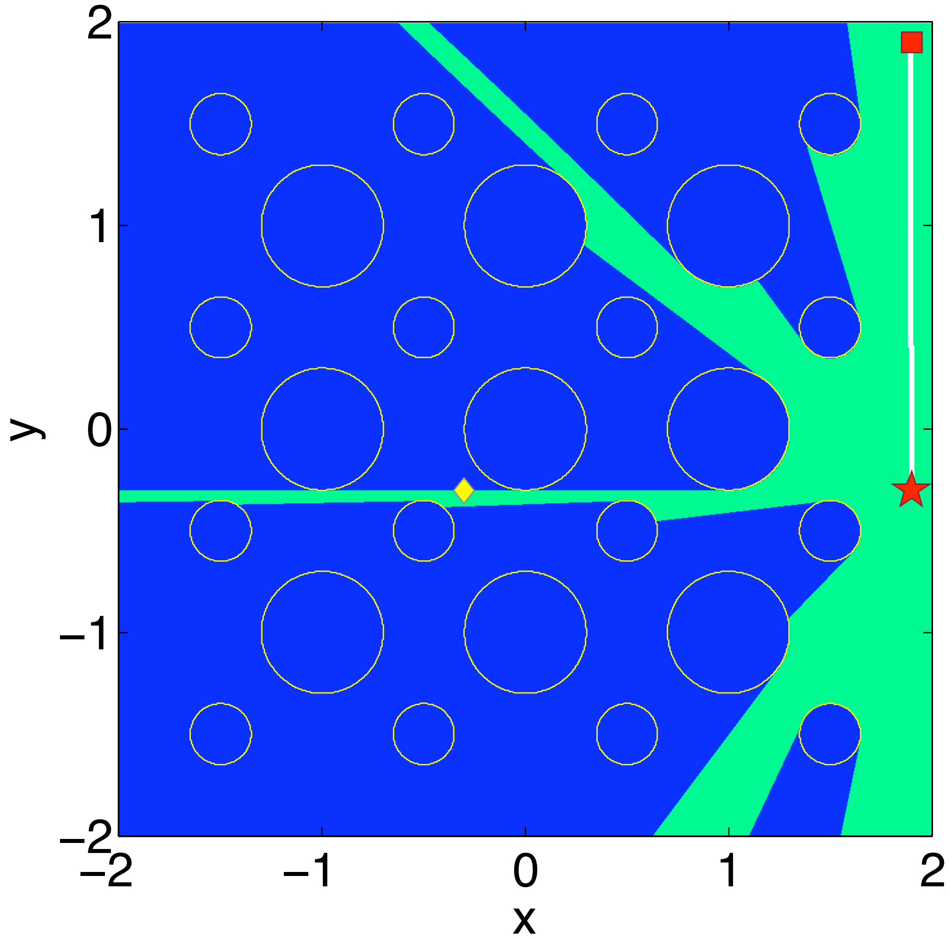
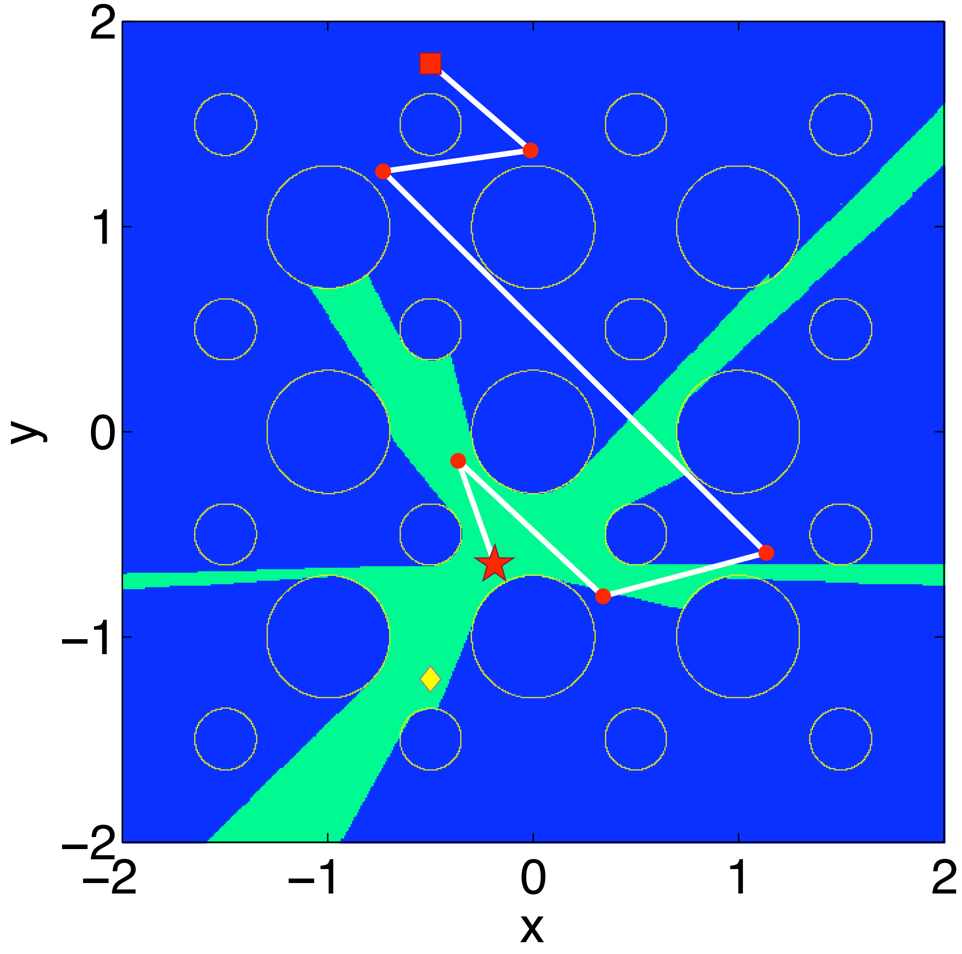
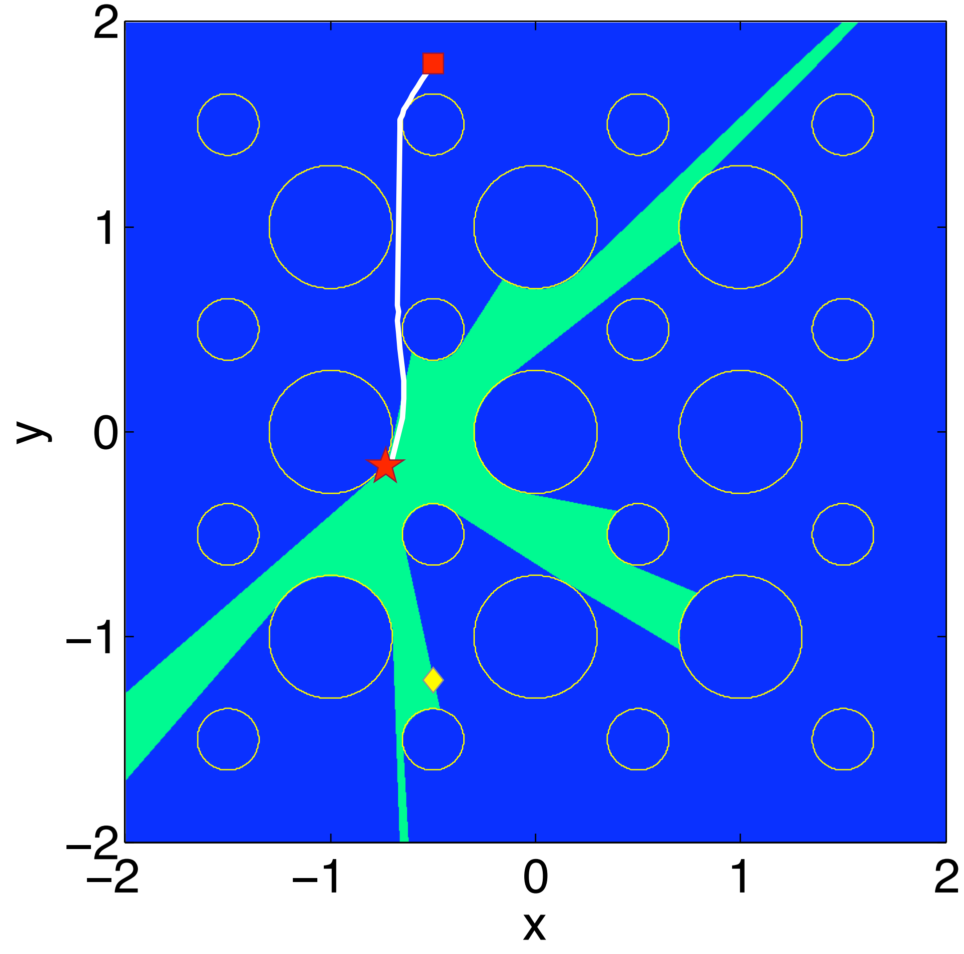




Shortest path if obstacles are given.

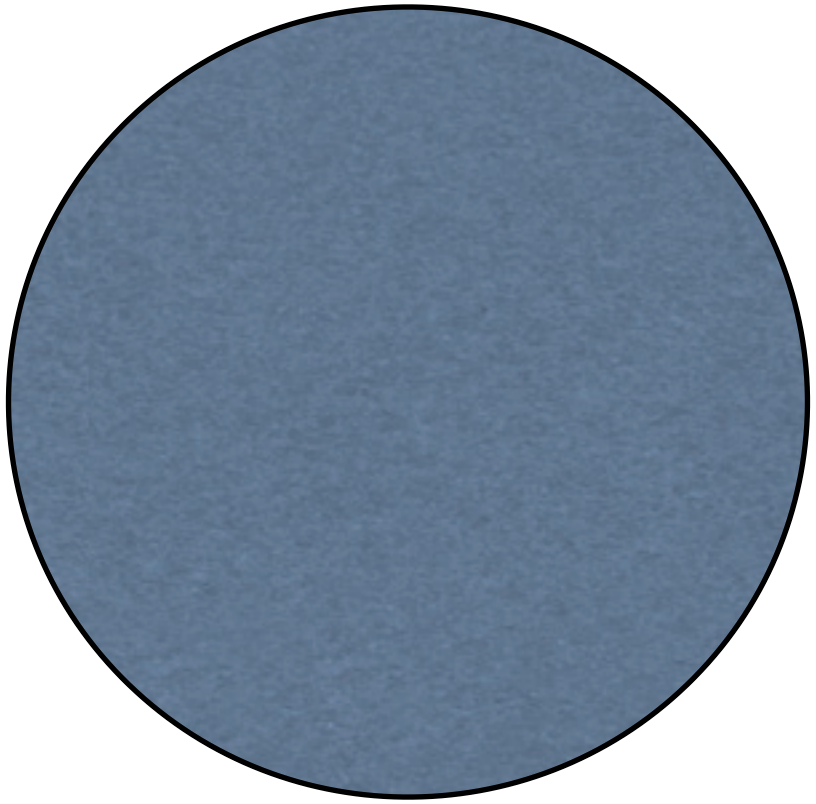




Shortest path when obstacles are known.

Where is the cheese?

O



S ?

Info available:

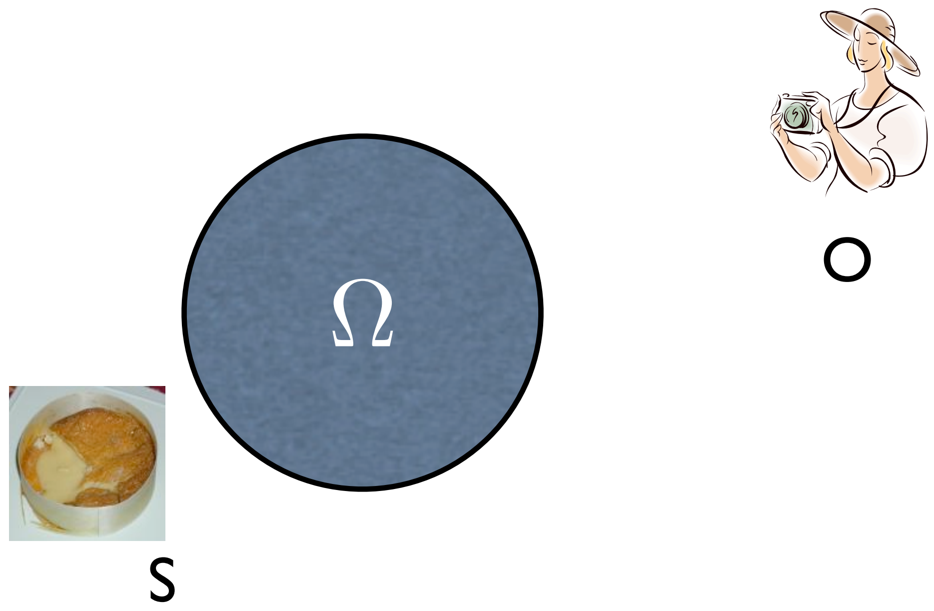
- Obstacles
- visibility $\phi(\cdot, O)$
- smell: $u(O,S),\ \nabla_1 u(O,S)$
  (intensity, gradient)

Reciprocity

Self-adjoint problem

$$-\Delta u = \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

(how the cheese smells)



Reciprocity

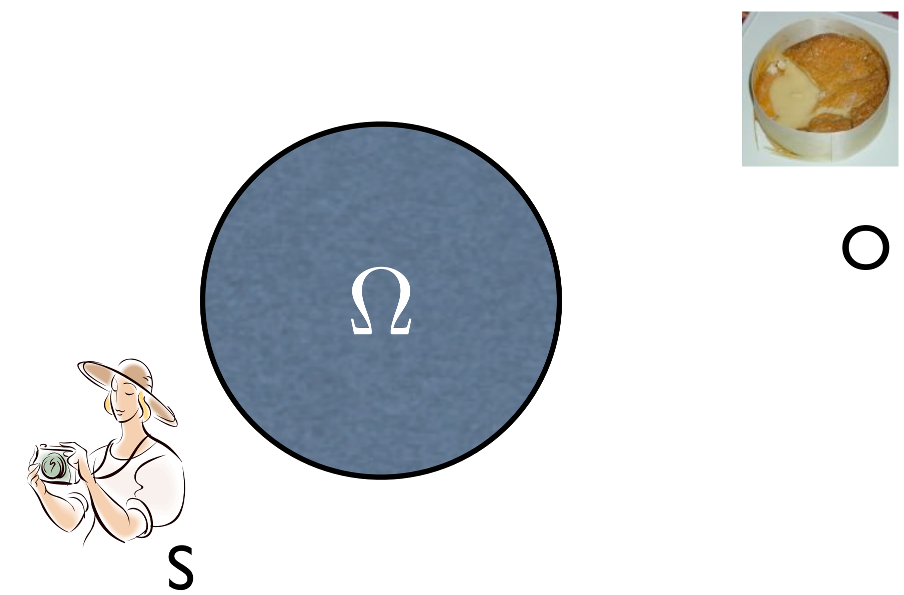
Self-adjoint problem

$$-\Delta u = \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

(how the cheese smells)

$$-\Delta v = \delta(x - O), \quad v|_{\partial\Omega} \equiv 0$$

(how my cheese smells)



Reciprocity

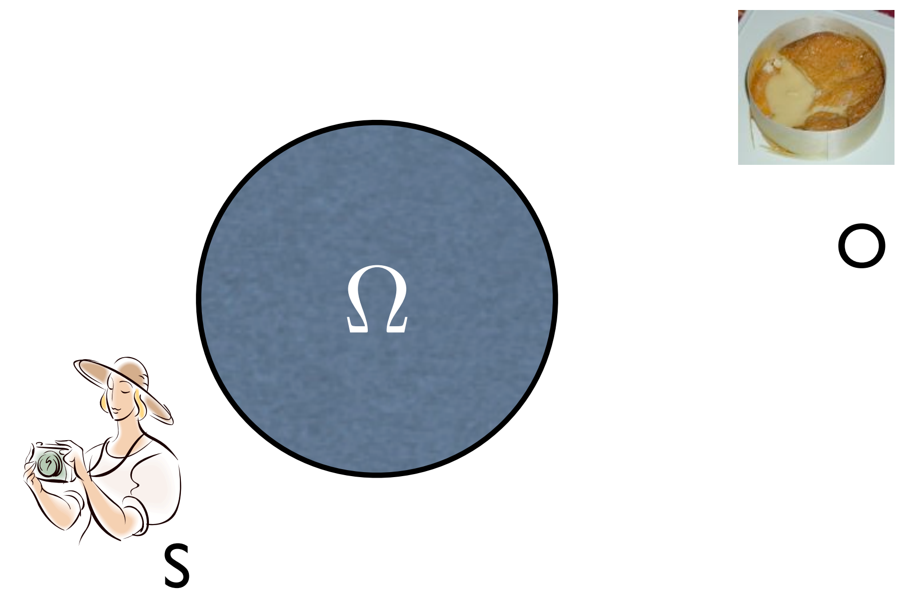
Self-adjoint problem

$$-\Delta u = \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

(how the cheese smells)

$$-\Delta v = \delta(x - O), \quad v|_{\partial\Omega} \equiv 0$$

(how my cheese smells)



$$-\int_\Omega \Delta u \; v \quad dx \quad = \quad -\int_\Omega u \; \Delta v \quad dx$$

$$v(S) \quad = \quad u(O)$$

$$u(x,y)|_{y=S} = u(x)$$

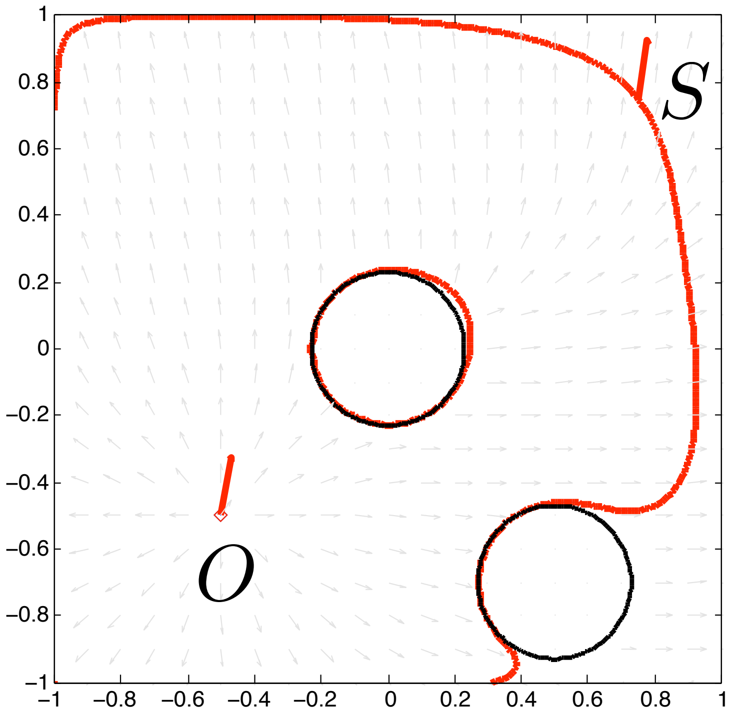
$$u(x,y) = u(y,x)$$

Search with complete knowledge of obstacles

Determine: source location (S)

$$-\Delta u = \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

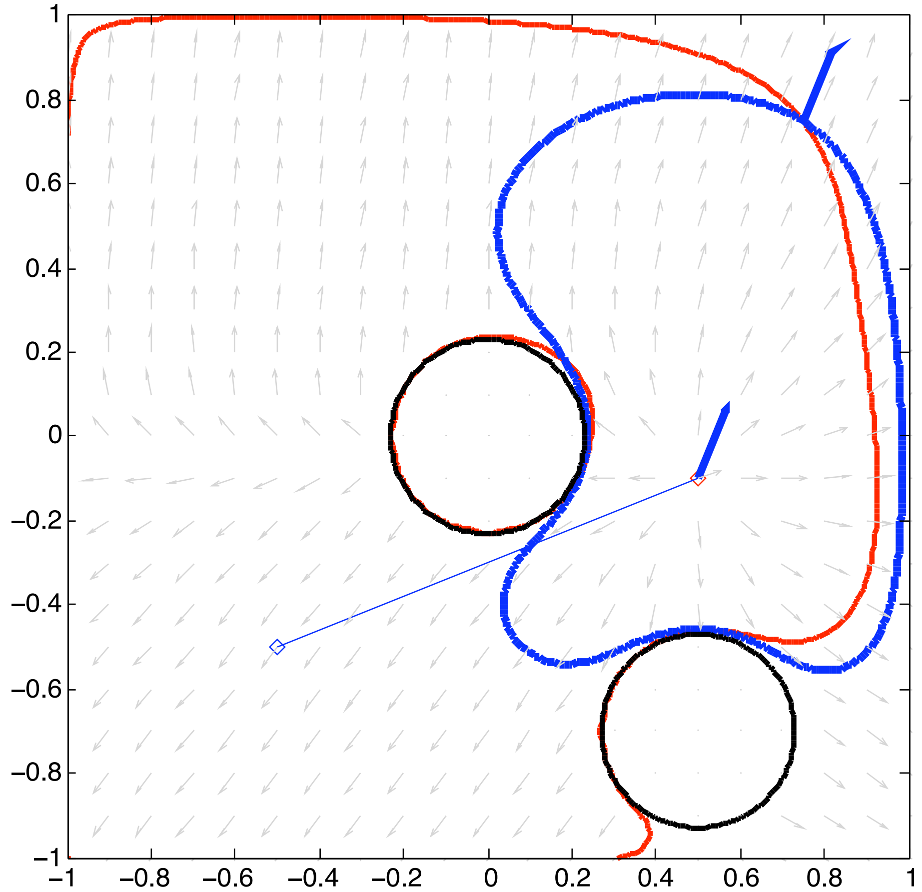
$$-\Delta v = \delta(x - O), \quad v|_{\partial\Omega} \equiv 0$$



$$v(S) = u(O) = I$$

given intensity

$$\Longrightarrow \; S \in \{v = I\}$$



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−0.2
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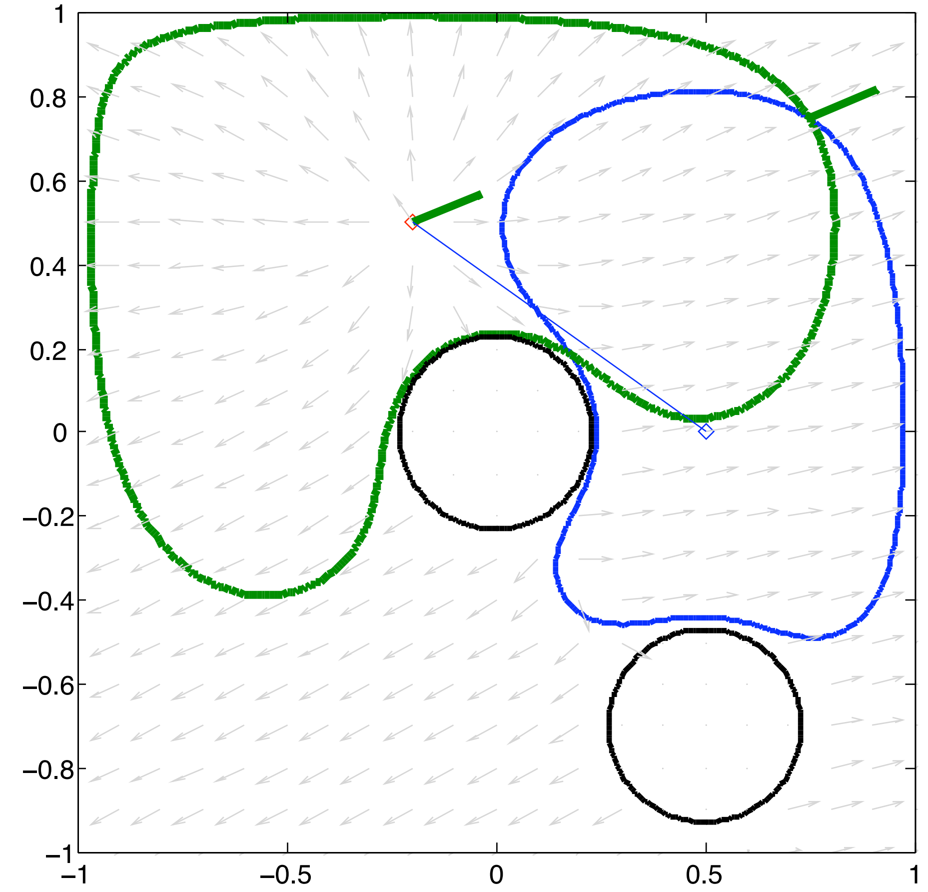
1



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−0.2
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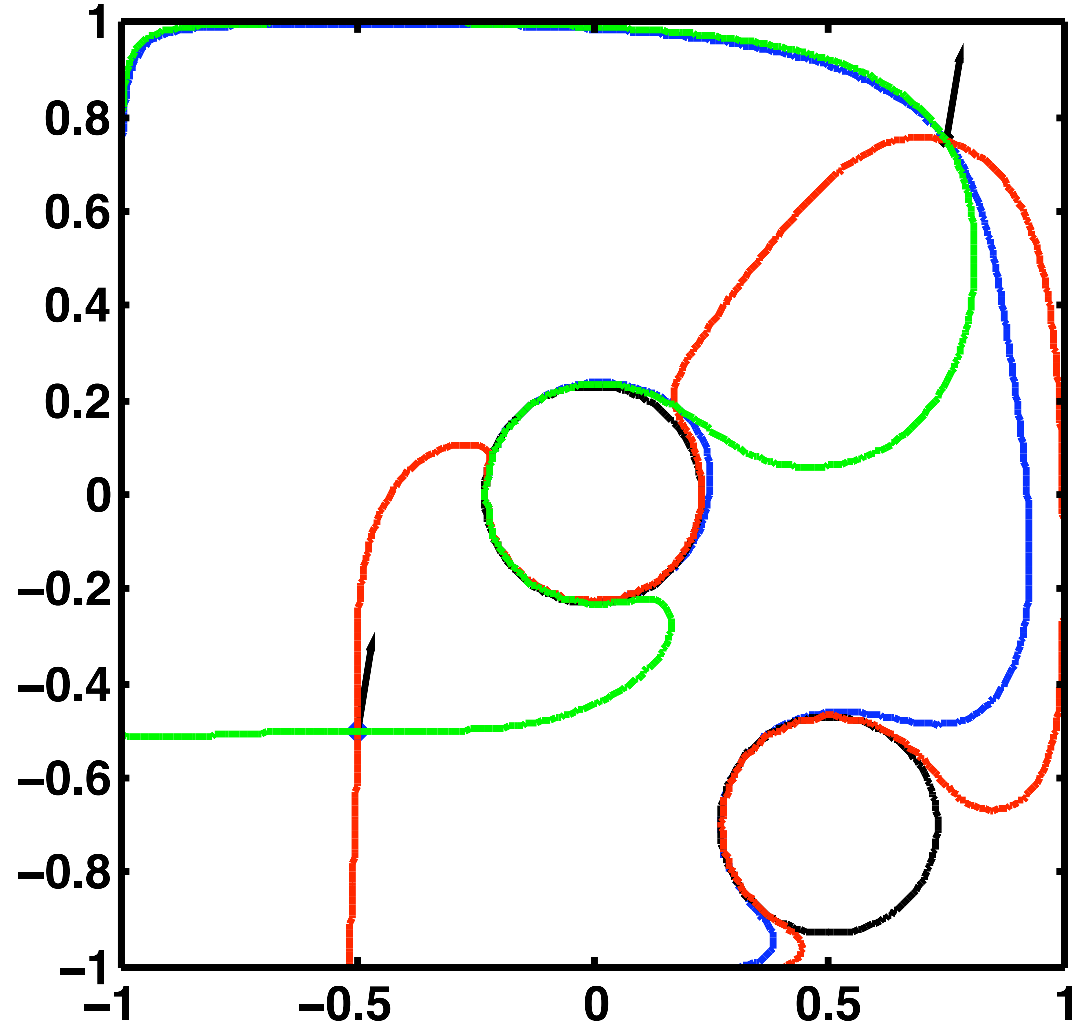
−0.6
−0.8
−1
−1
−0.5
0
0.5
1

Derivative information

$$\Delta w^{(j)} = \frac{\partial}{\partial x_j}\delta(x - O), \quad w^{(j)}|_{\partial\Omega} \equiv 0$$

$$\Longrightarrow \; w^{(j)}(S) = u_{x_j}(O)$$



Confidence function

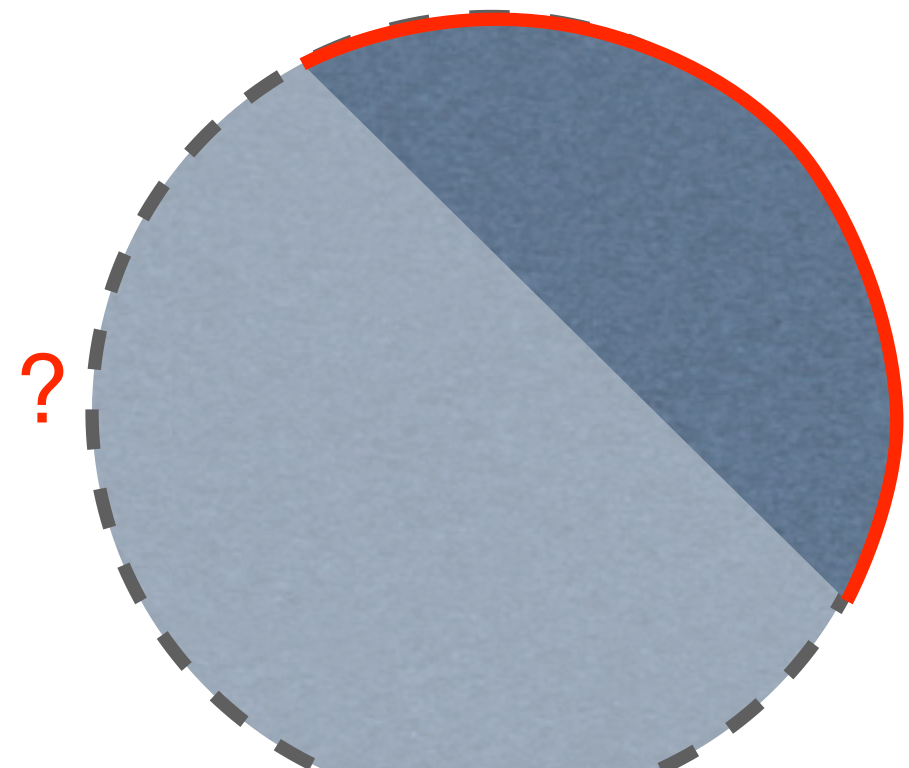
$$u_k := u(O_k), \quad p_k := u_x(O_k), \quad q_k := u_y(O_k)$$

$$h_k(x) \quad := \quad e^{-\alpha(v_k(x)-u_k)^2} + c_0(e^{-\beta(w_k^{(1)}(x)-p_k)^2} + e^{-\beta(w_k^{(2)}(x)-q_k)^2})$$

$$H(x) = \sum h_k(x)$$

Partial visibility information on obstacles

O

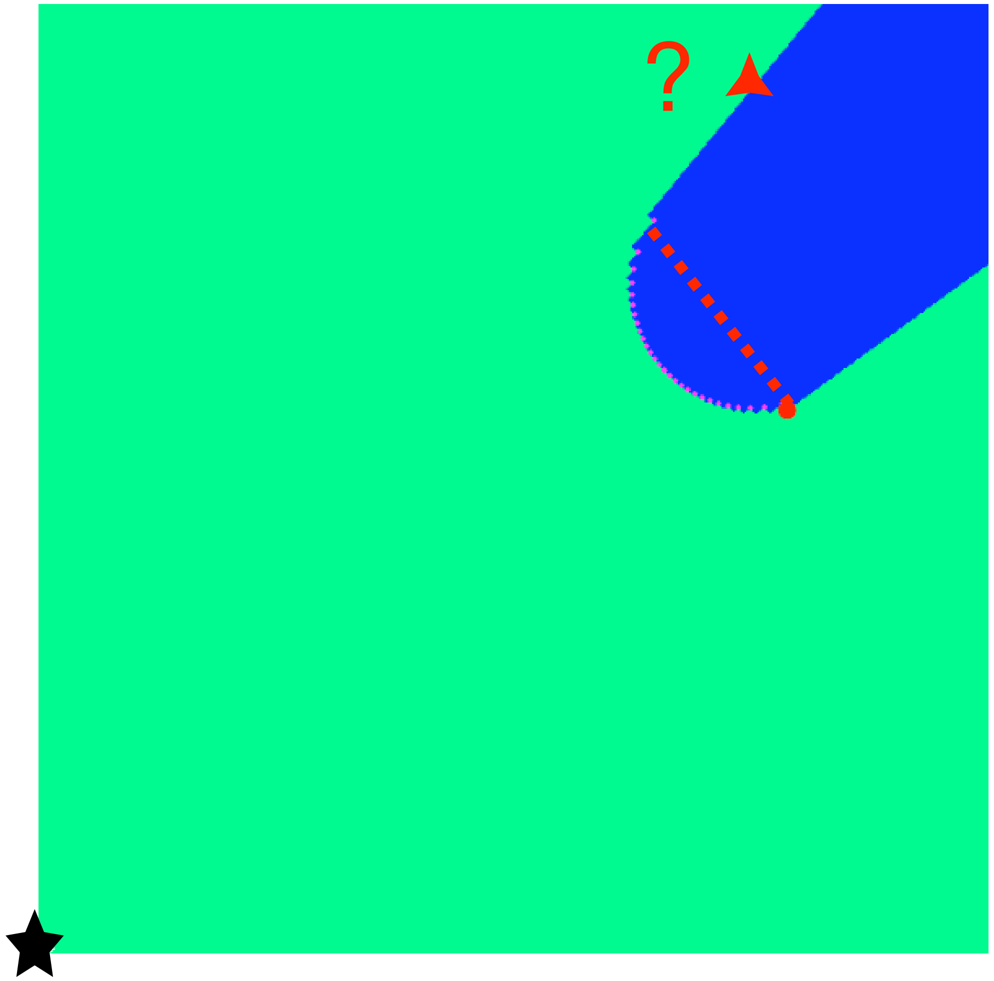


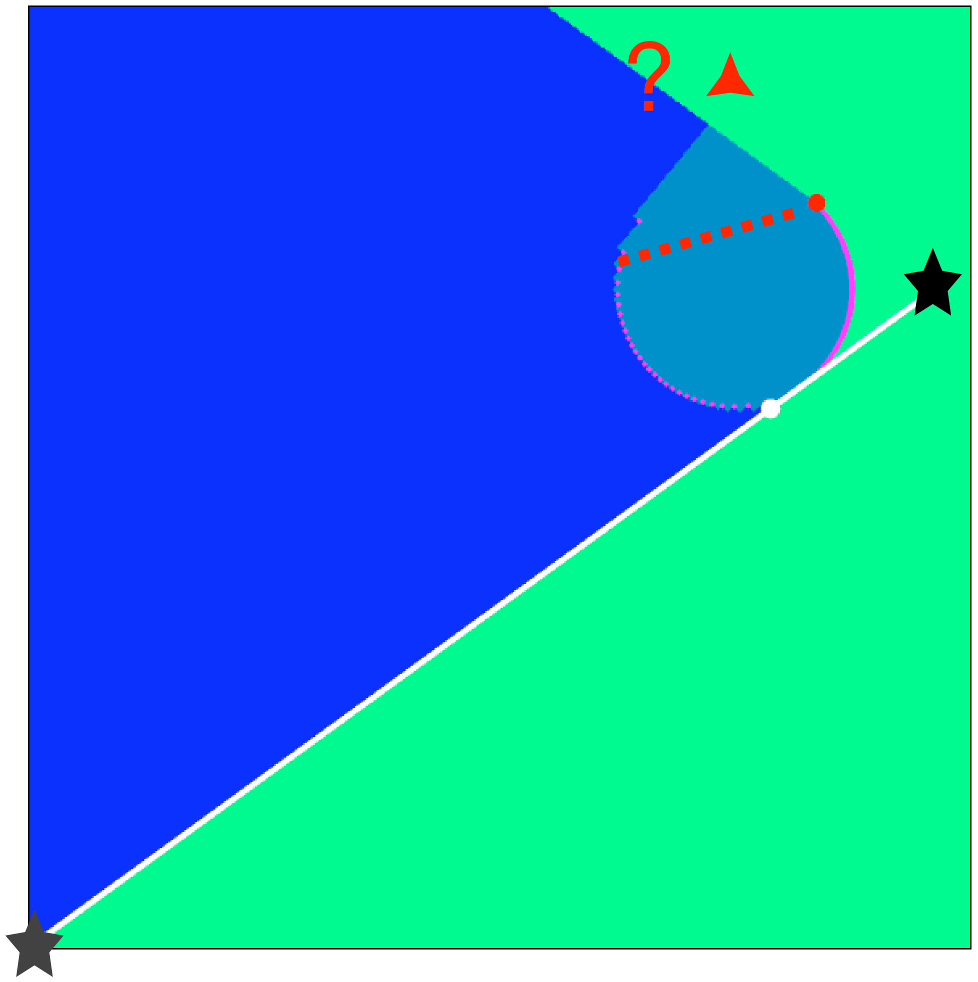
S ?

Info available:

- visibility: $\phi(\cdot, O)$
- smell: $u(O,S),\ \nabla_1 u(O,S)$
  (intensity, gradient)

Adaptively increase the knowledge on the obstacle, and source location





Comparison principle

$$-\Delta v = \delta(x - O) \;\; \text{in } \Omega^c, \quad v|_\Omega \equiv 0$$

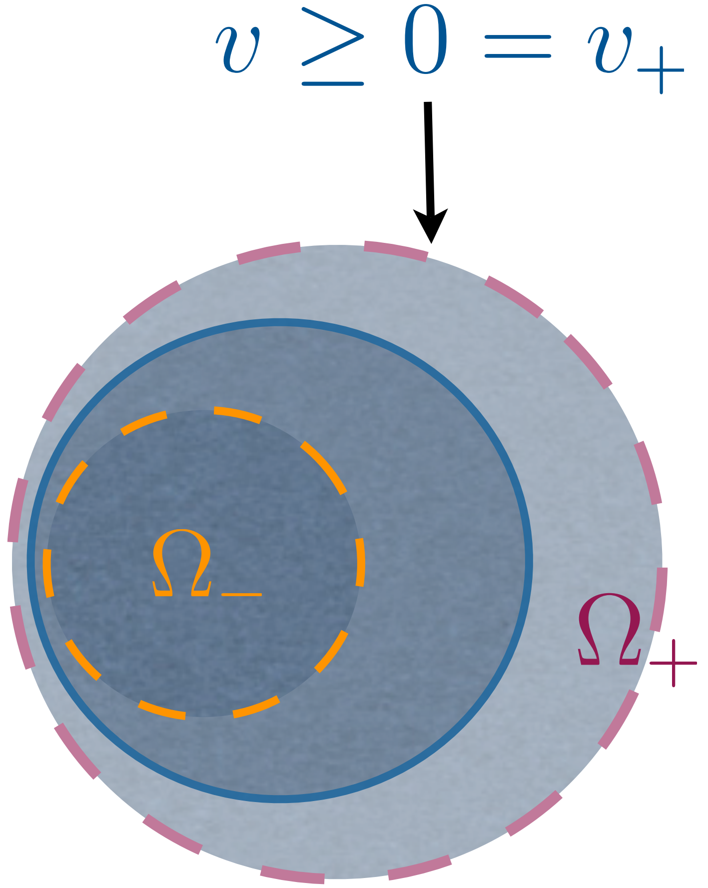
$$-\Delta v_\pm = \delta(x - O) \;\; \text{in } \Omega^c_\pm, \quad \tilde{v}_\pm|_{\Omega^c_\pm} \equiv 0$$

$$\Omega_- \subset \Omega \subset \Omega_+$$

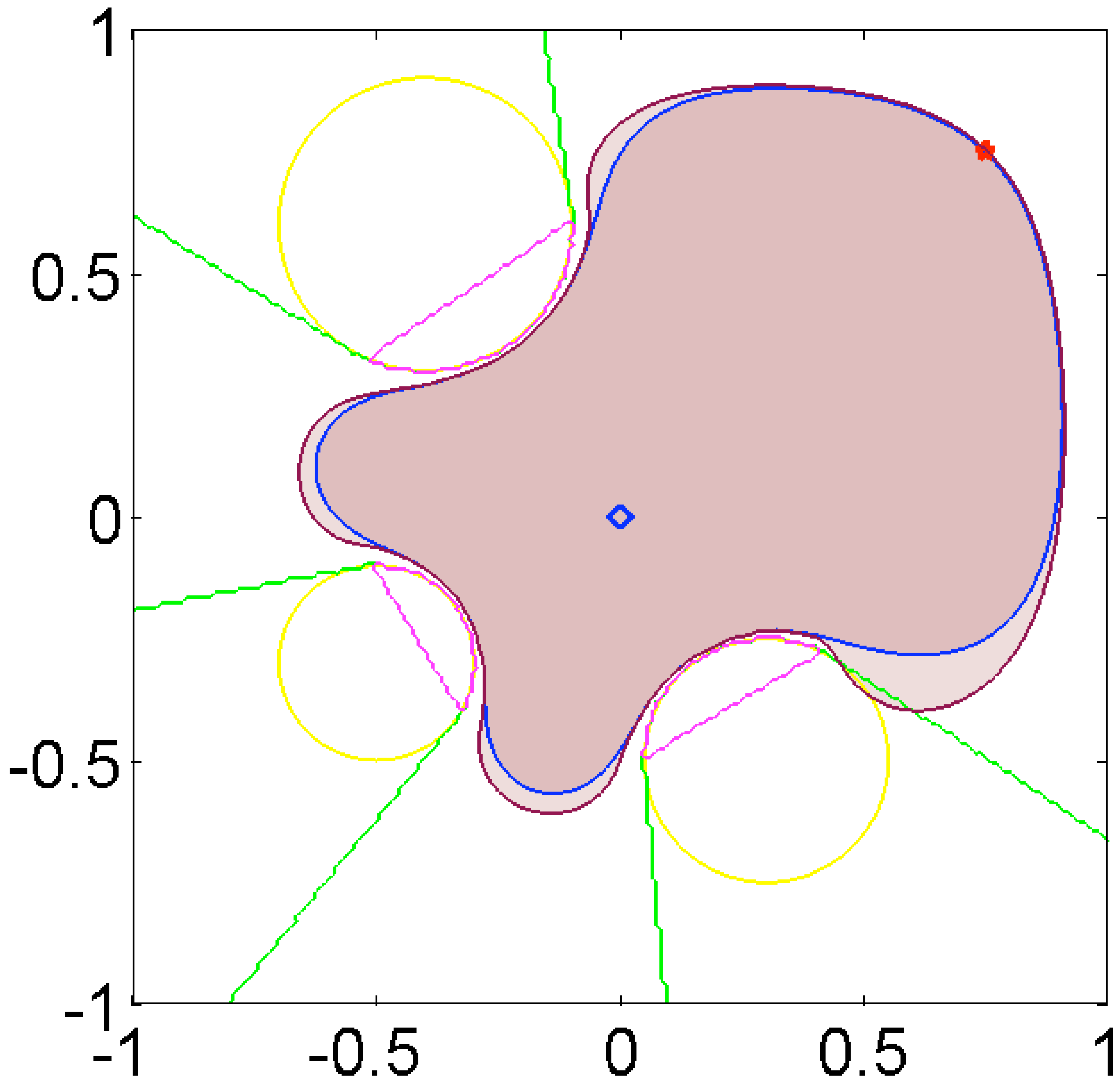
$$\implies \; v_- \geq v \geq v_+$$

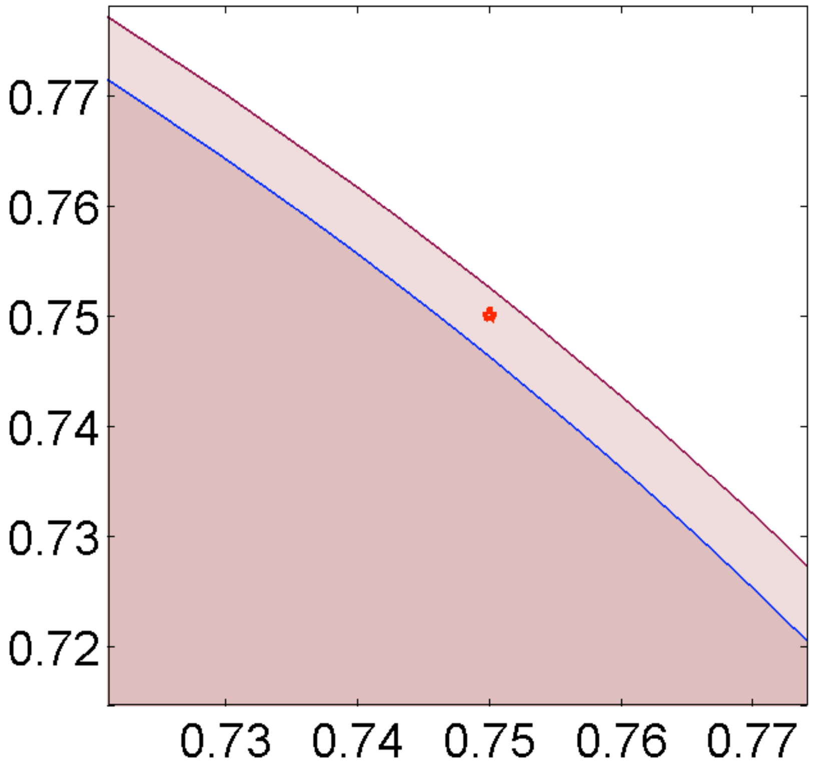
$$I = u(O) = v(S) \quad \leq \quad v_-(S)$$

$$\geq \quad v_+(S)$$



$$i.e. \quad S \in \{x : v_-(x) \geq I \geq v_+(x)\}$$

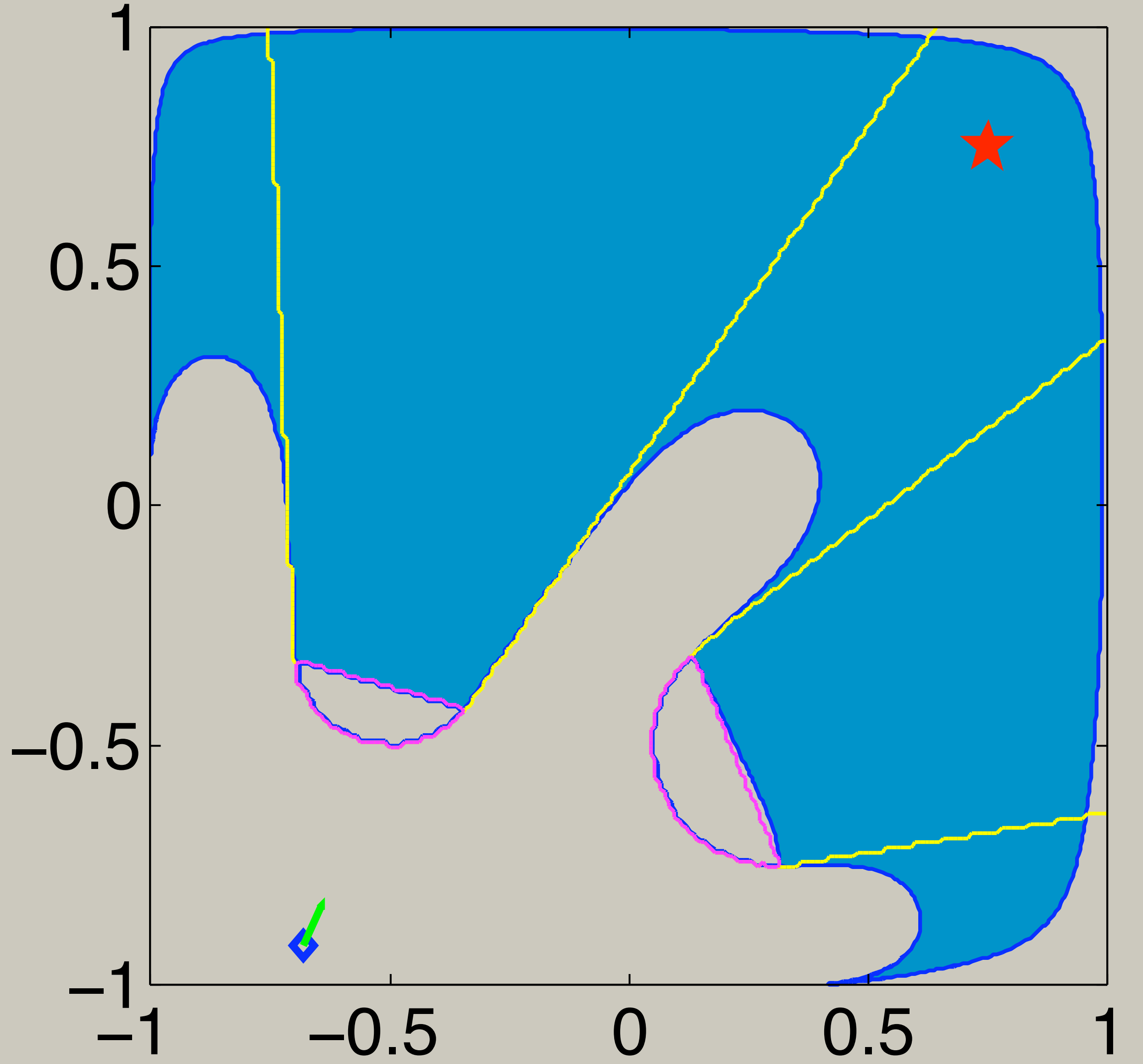


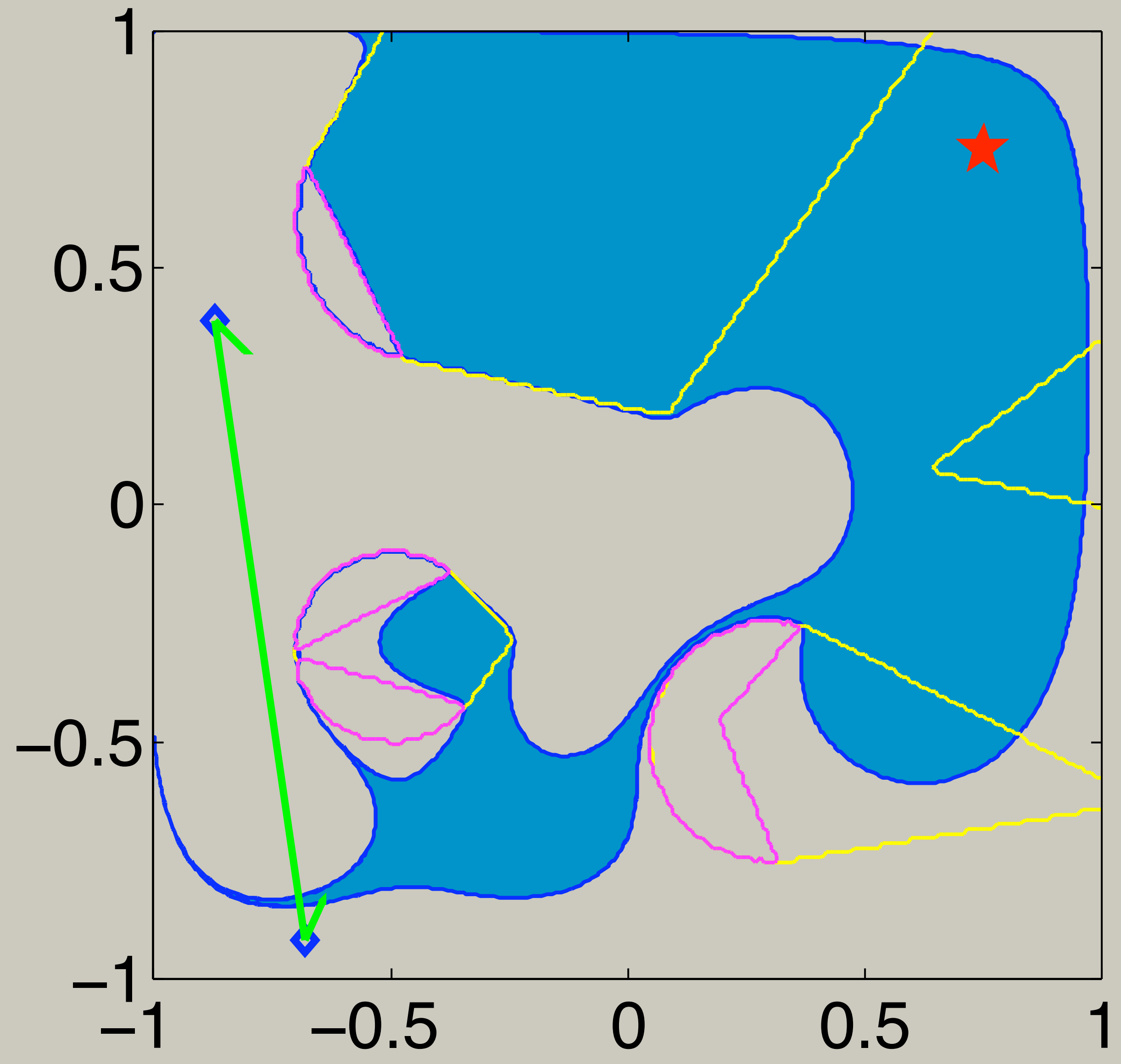


$$S \in \{x : v_-(x) \geq I \geq v_+(x)\}$$

Sandwiching by under and over approx. obstacles $\Omega_- \subset \Omega \subset \Omega_+$

Search with increasing knowledge of obstacles from vis. information

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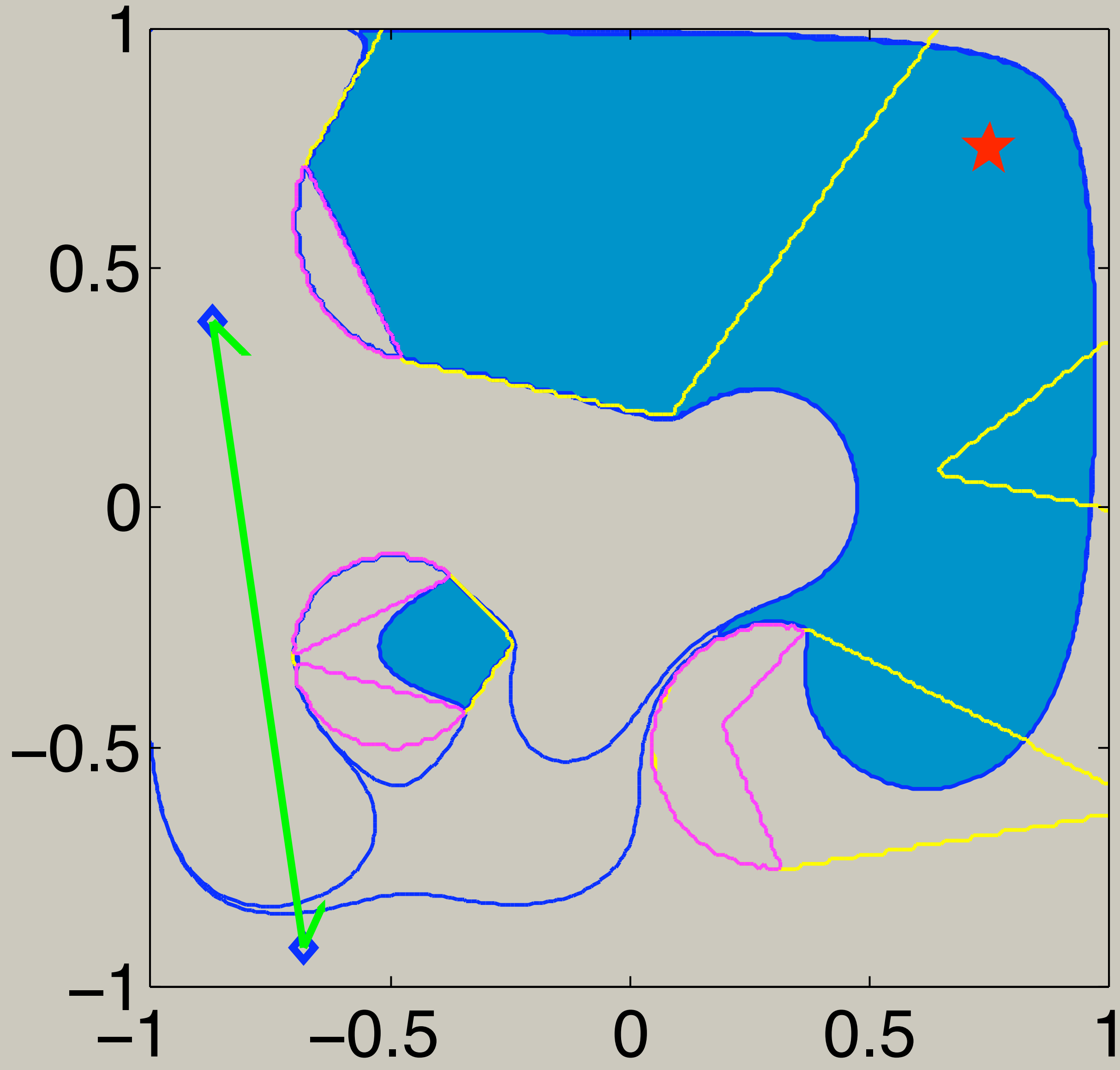
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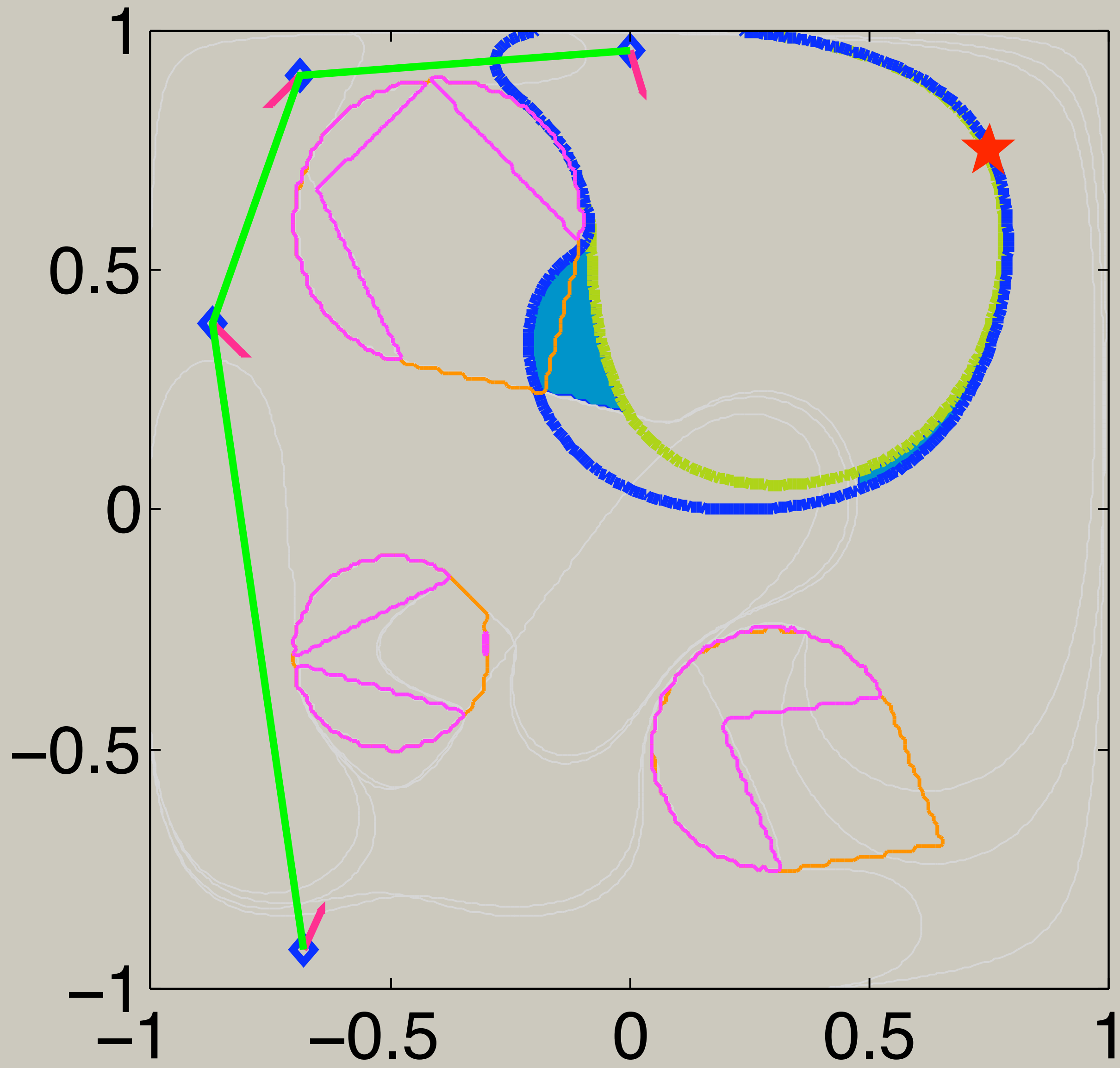
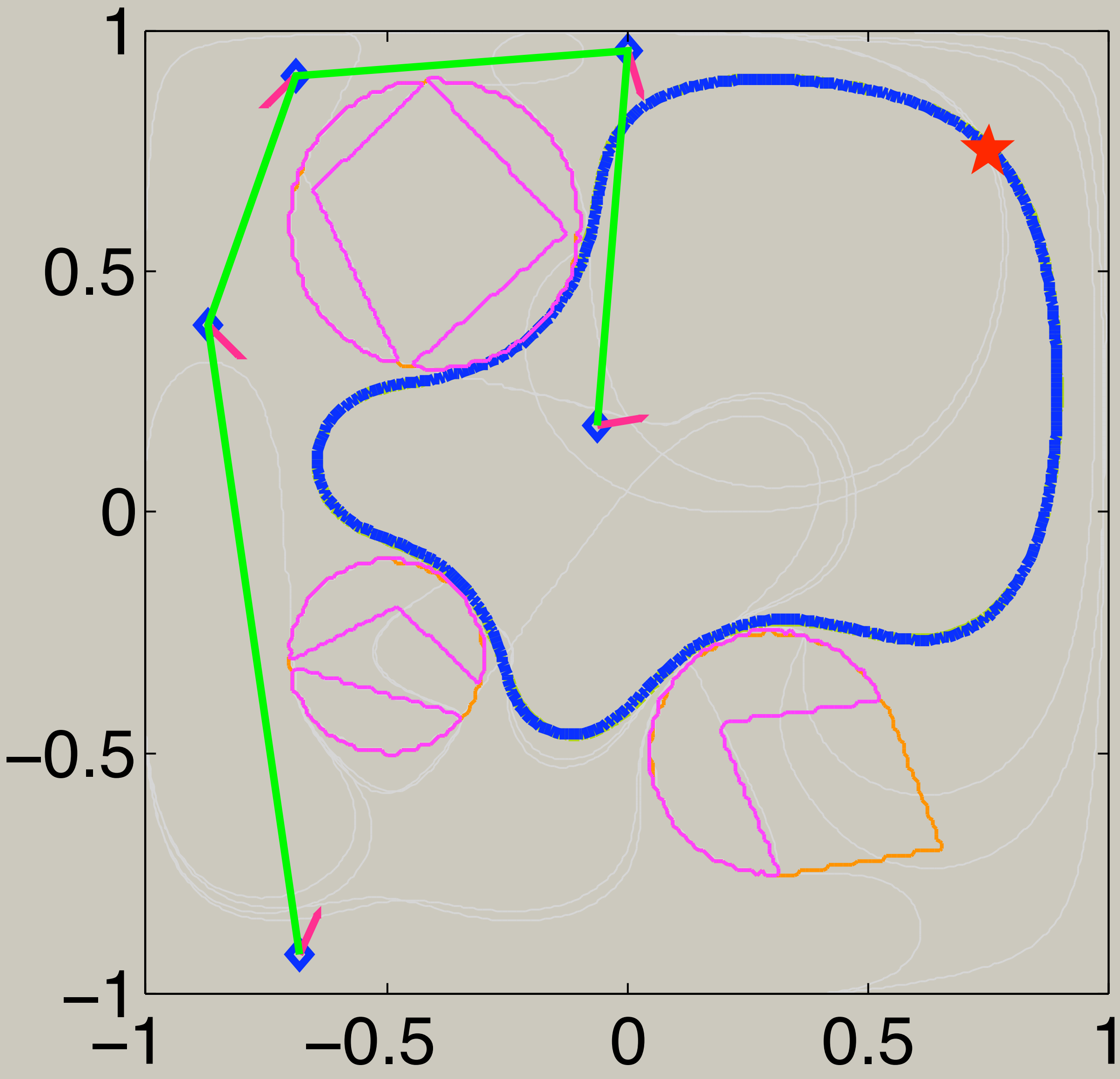
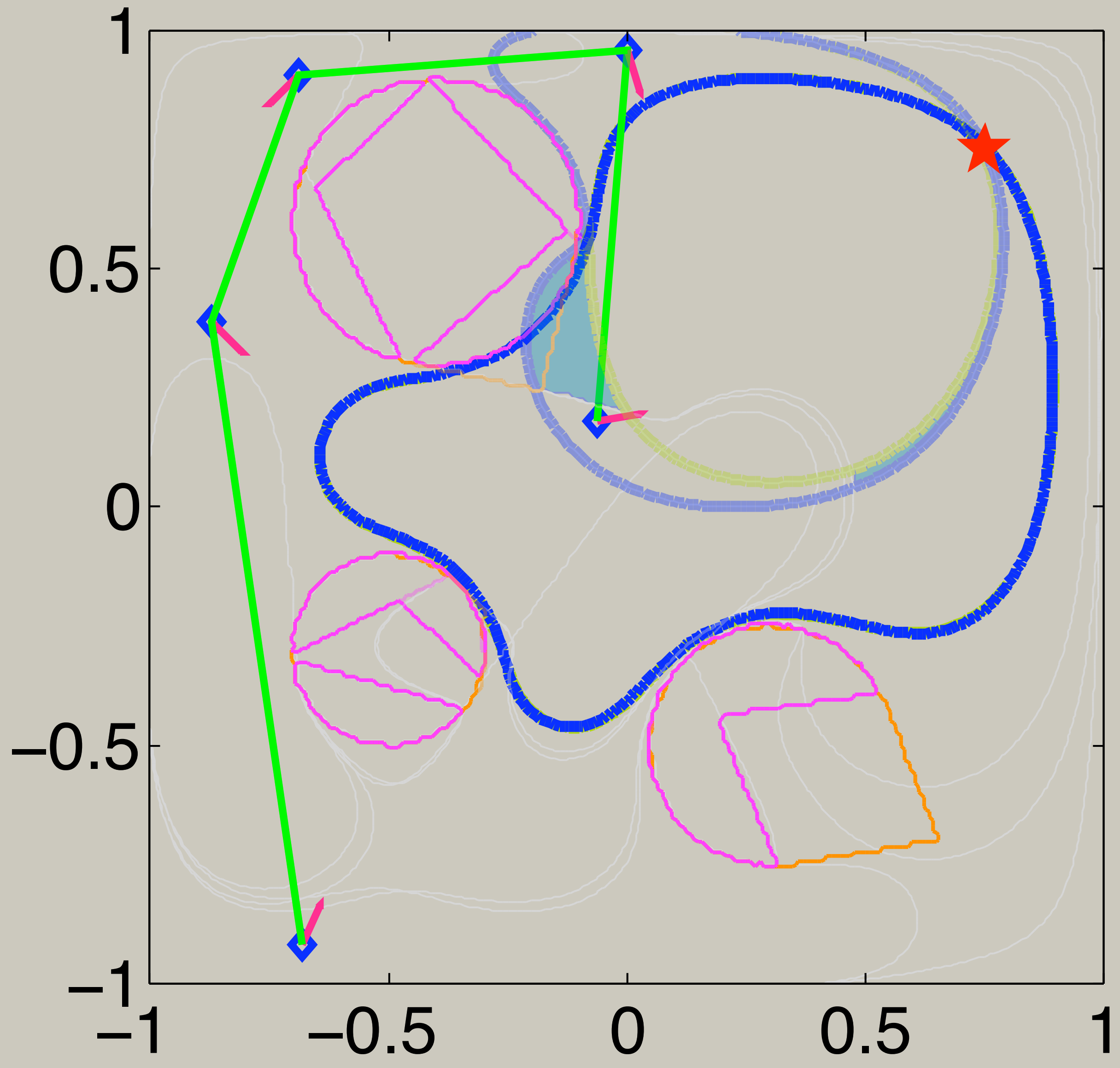
Search with increasing knowledge of obstacles from vis. information





Search terminates when
the region of possible source location is visible
at current vantage location.



Search terminates when
the region of possible source location is visible
at current vantage location.

Single source of unknown strength

Complete knowledge of the obstacle.

$$-\Delta u = \alpha \; \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

$$-\Delta v_j = \delta(x - O_j), \quad v|_{\partial\Omega} \equiv 0, \quad j = 1, 2$$

$$O_1 \neq O_2$$

$$\begin{aligned} \alpha \; v_1(S) &= u(O_1) \\ \alpha \; v_2(S) &= u(O_2) \end{aligned} \quad \Longrightarrow \quad S \in \{x : \frac{v_1(x)}{v_2(x)} = \frac{u(O_1)}{u(O_2)}\}$$

Single source of unknown strength

Partial knowledge of the obstacle.

$$-\Delta u = \alpha \ \delta(x - S), \quad u|_{\partial\Omega} \equiv 0$$

$$-\Delta v_\pm = \delta(x - O) \ \text{ in } \Omega^c_\pm, \quad \tilde{v}_\pm|_{\Omega^c_\pm} \equiv 0$$

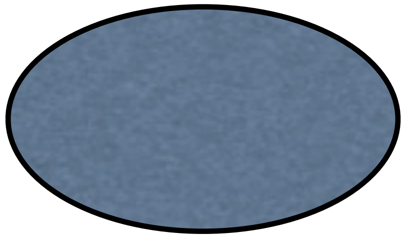
$$\Omega_- \subset \Omega \subset \Omega_+$$

$$\Longrightarrow \ v_- \geq v \geq v_+ \geq 0$$

$$\begin{aligned} \alpha \ v_-(S) &\geq u(O_1) \\ \alpha \ v_+(S) &\leq u(O_2) \end{aligned} \quad \Longrightarrow \quad S \in \{x : \frac{v_-(x)}{v_+(x)} \geq \frac{u(O_1)}{u(O_2)}\}$$

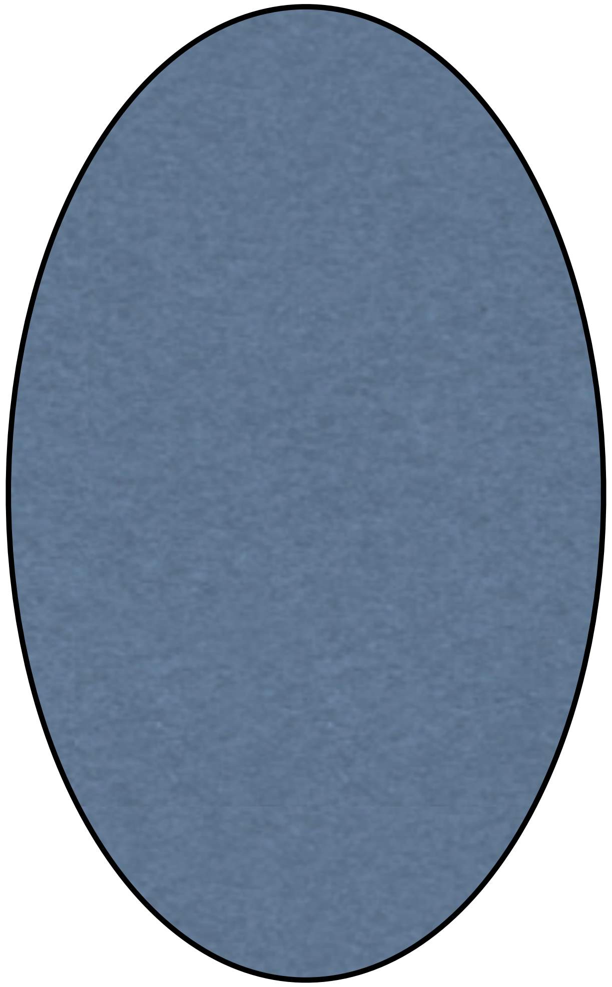
Where are the cheeses?

S ?



$$-\Delta u = \sum \delta(x - S_j), \quad u|_{\partial\Omega} \equiv 0$$

O



S ?

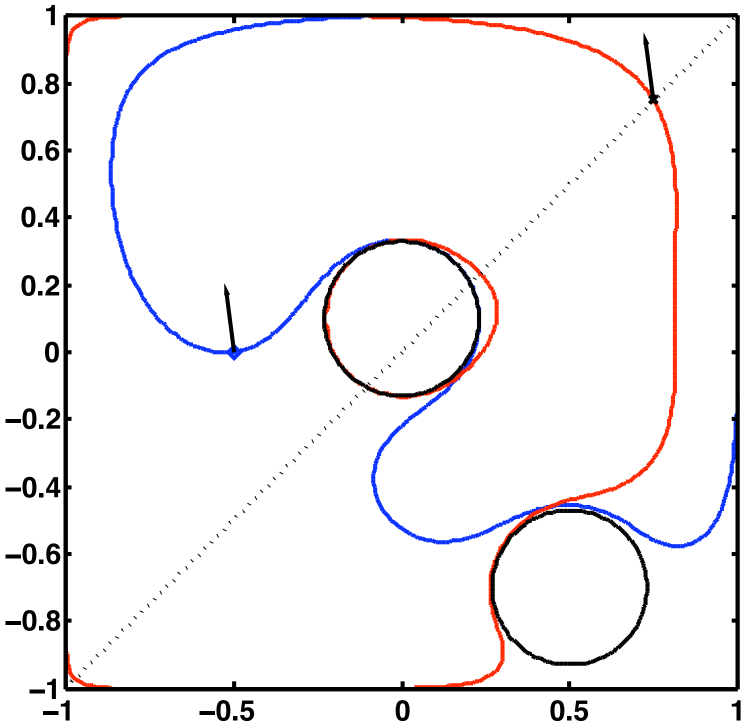
Info available:

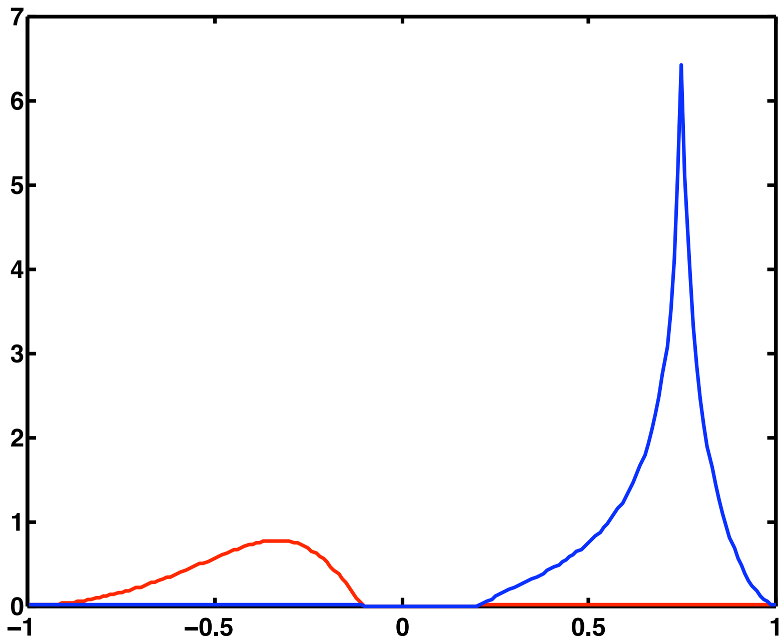
- Obstacles
- visibility $\phi(\cdot, O)$
- smell: $u(O, S)$, $\nabla_1 u(O, S)$
  (intensity, gradient)

Multiple sources and theirs decays

$$-\Delta u_j = \delta(x - S_j), \quad u_j|_{\partial\Omega} \equiv 0 \qquad u = \sum_j u_j$$

$$|O - S_1| \ll | O - S_j|, \quad j \neq 1 \qquad u_1(O) \sim \sum u_j(O)$$



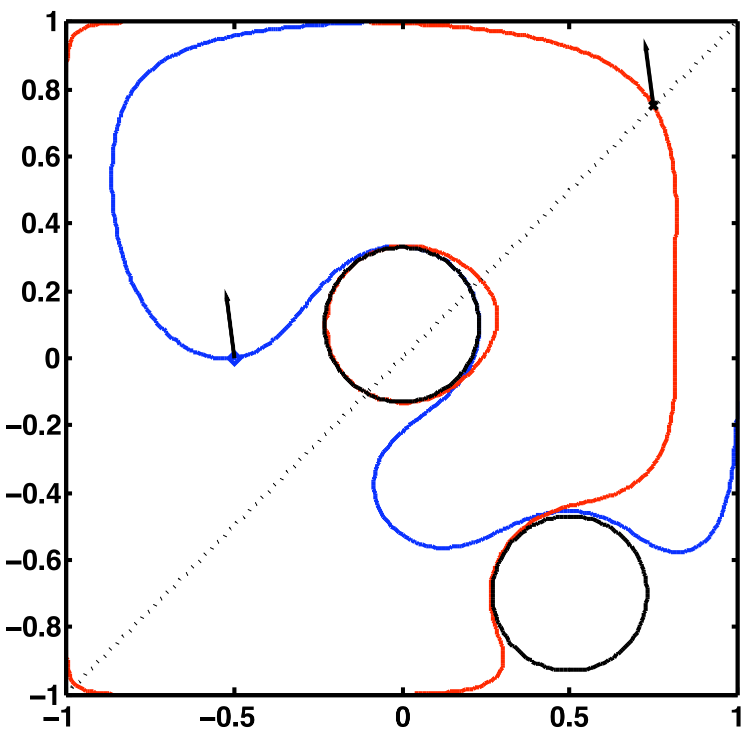


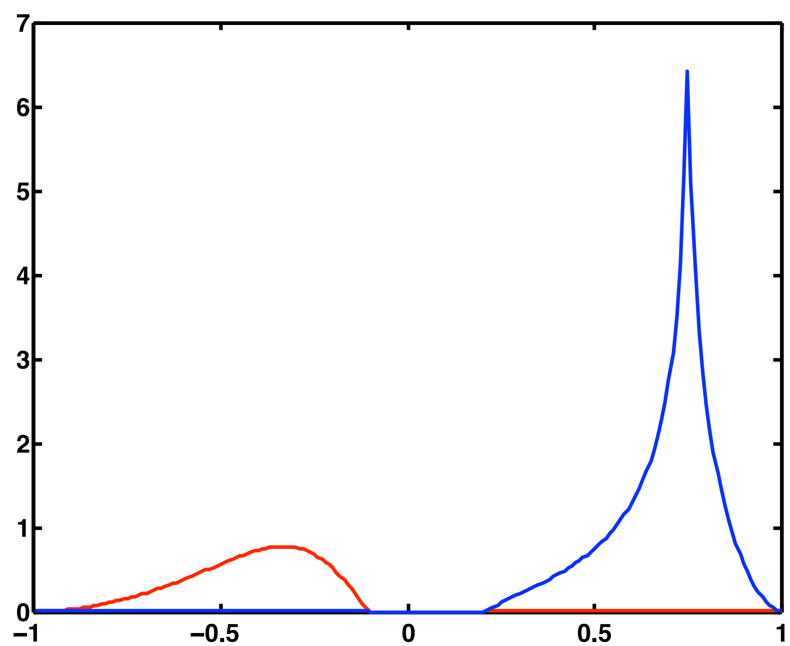
Multiple sources and theirs decays

$$-\Delta u_j = \delta(x - S_j), \quad u_j|_{\partial\Omega} \equiv 0 \qquad u = \sum_j u_j$$

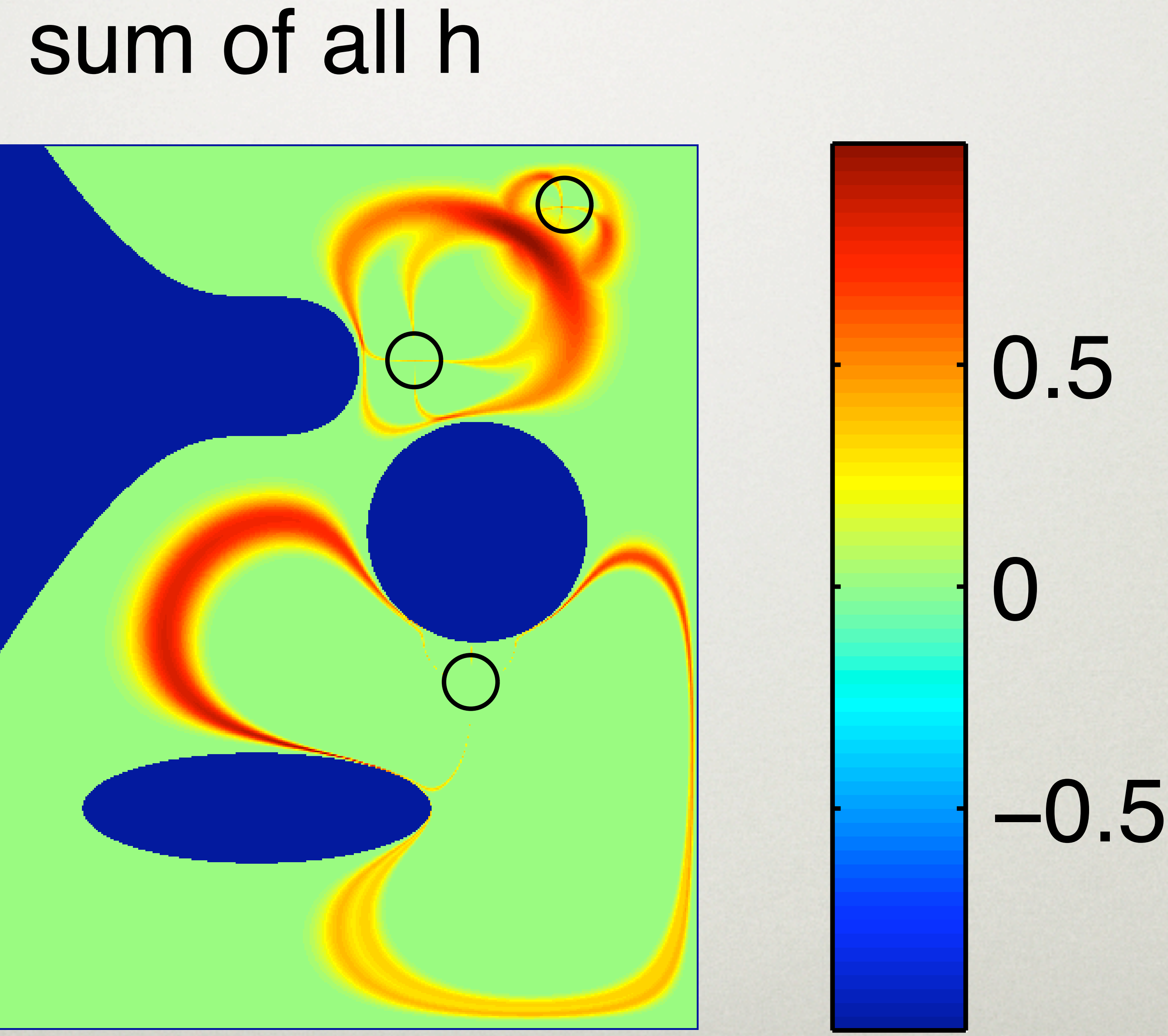
$$|O - S_1| \ll |O - S_j|, \quad j \neq 1 \qquad u_1(O) \sim \sum u_j(O)$$

Effective source ~ one of the sources

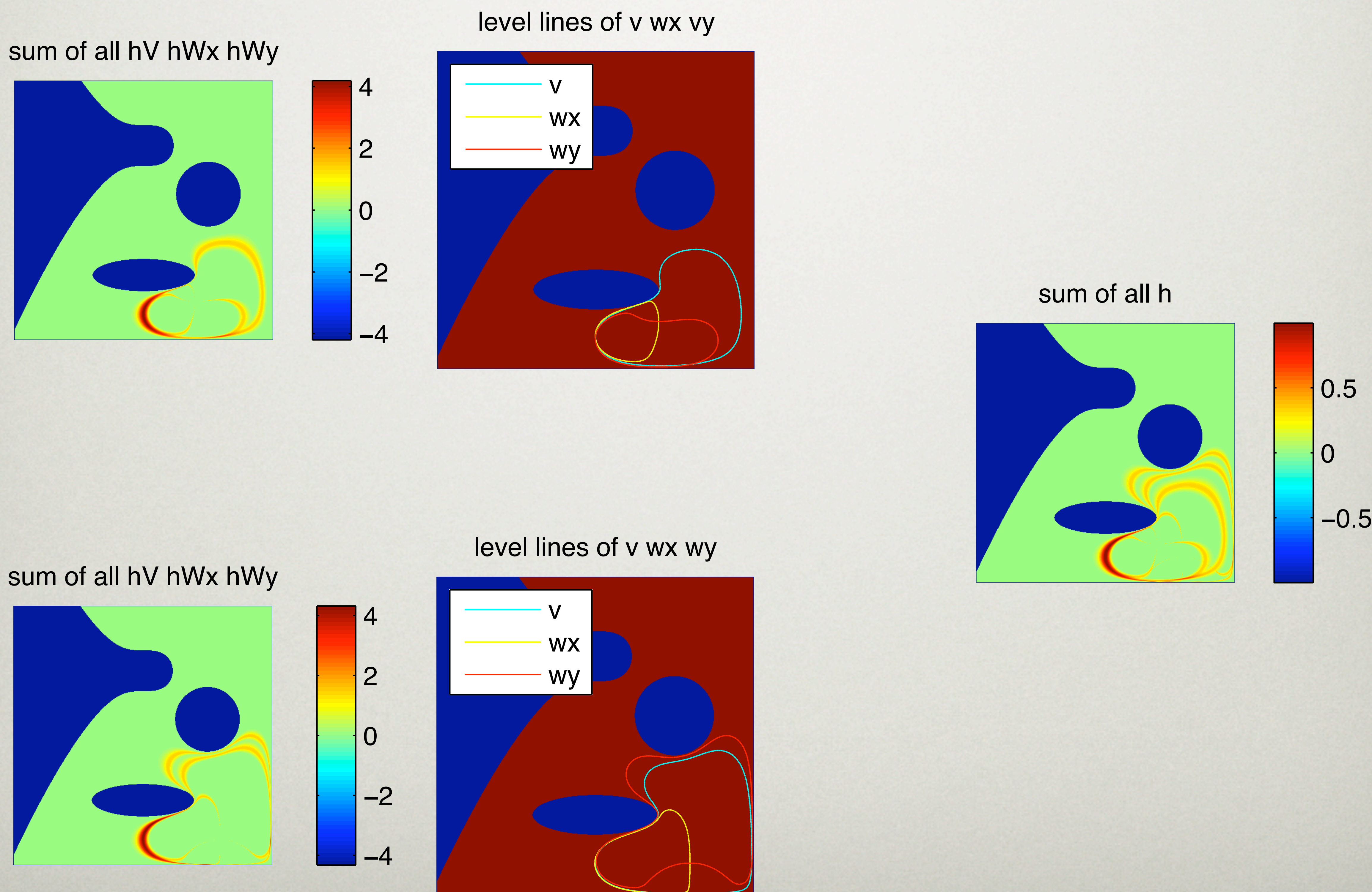




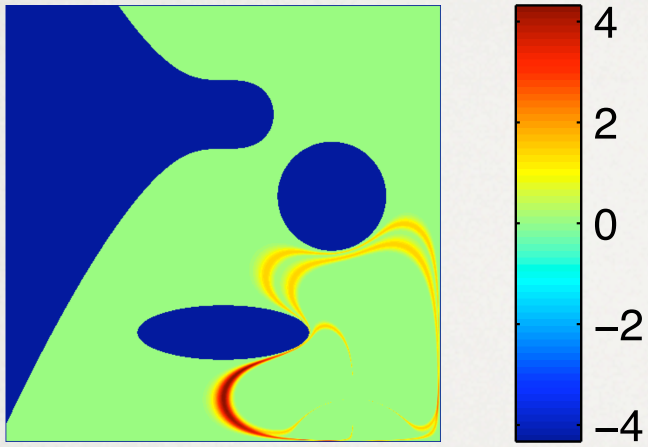
Inconsistent information from sensors



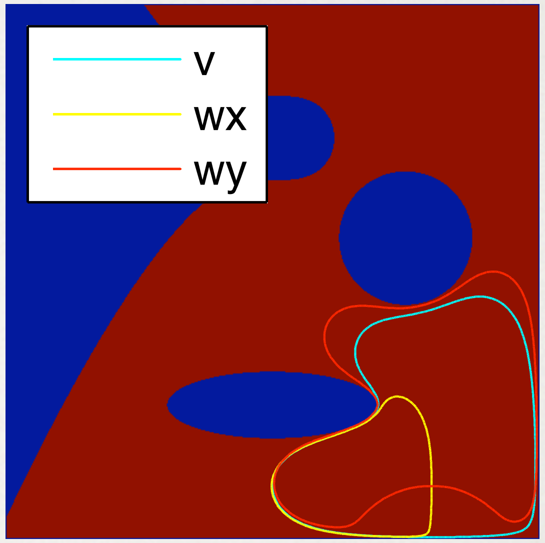
Consistent information from sensors

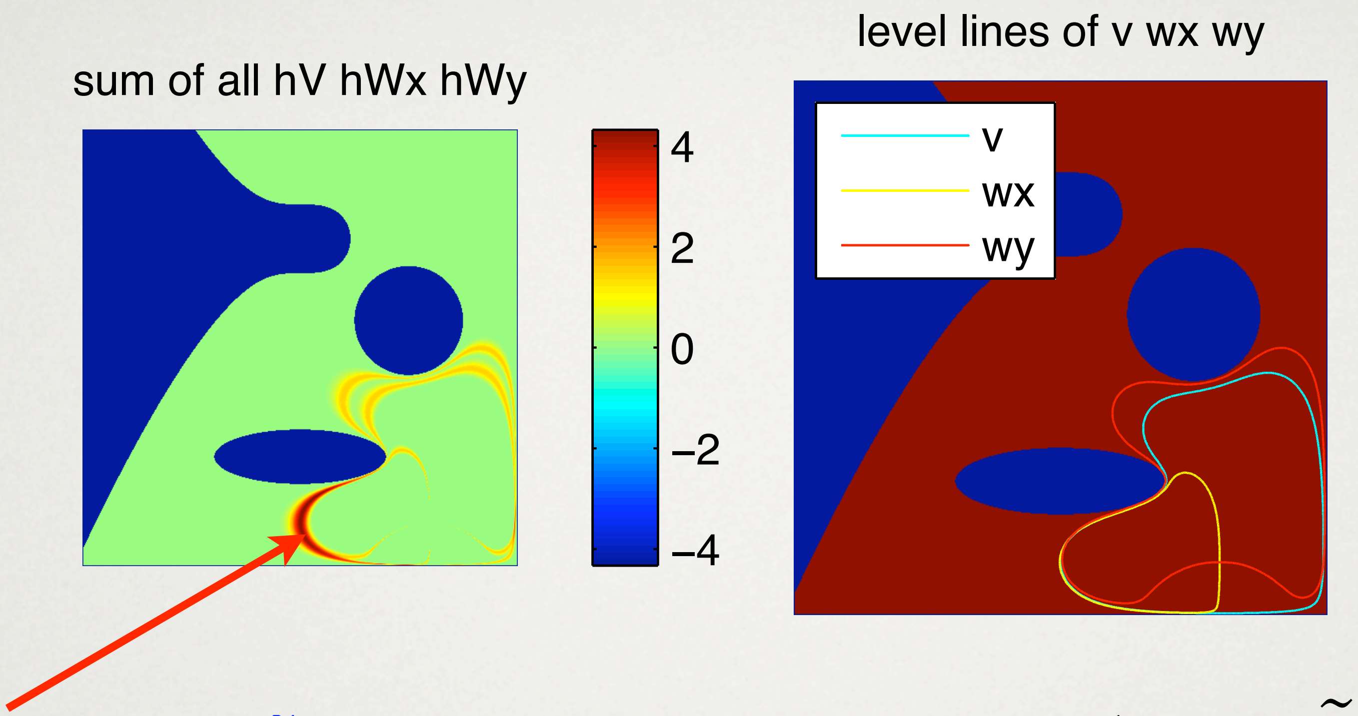


sum of all hV hWx hWy



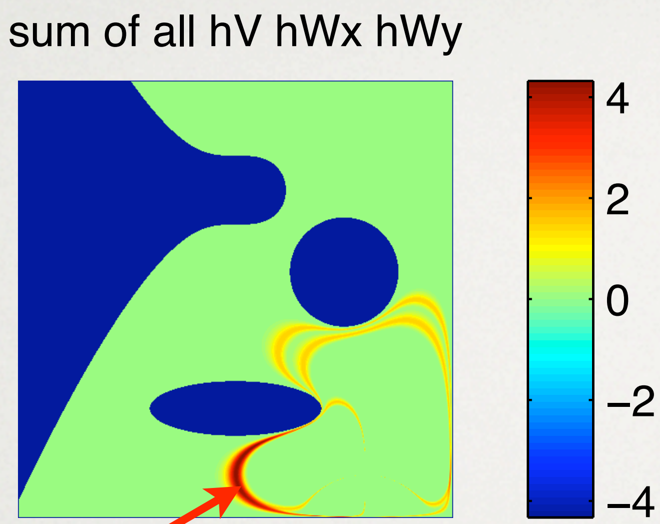
level lines of v wx wy

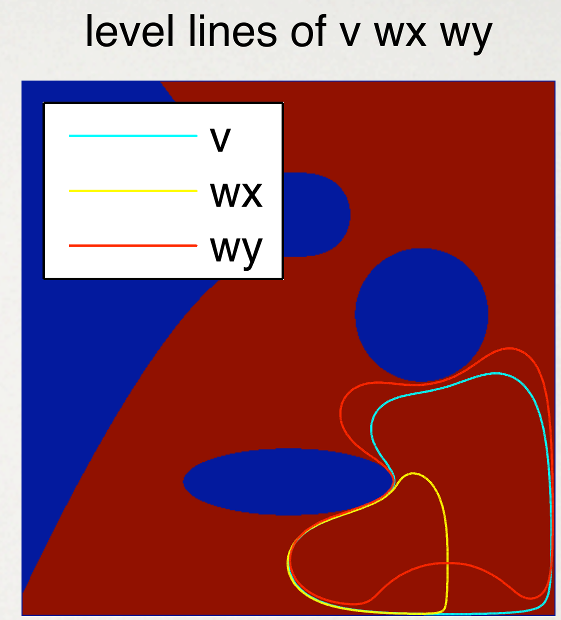




Identified souce: $\tilde{S}_1$

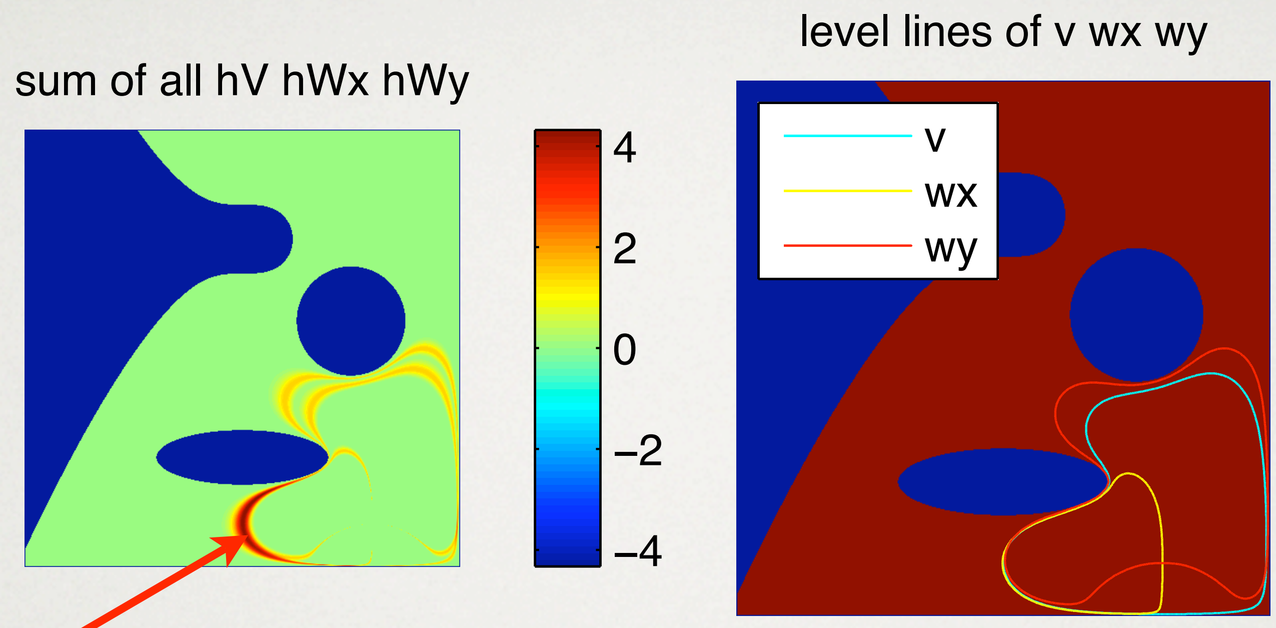
$$-\Delta \tilde{u}_1 = \delta(x - \tilde{S}_1), \quad \tilde{u}_j|_{\partial\Omega} \equiv 0$$





Identified souce: $\tilde{S}_1$ $\quad -\Delta \tilde{u}_1 = \delta(x - \tilde{S}_1), \quad \tilde{u}_j|_{\partial\Omega} \equiv 0$

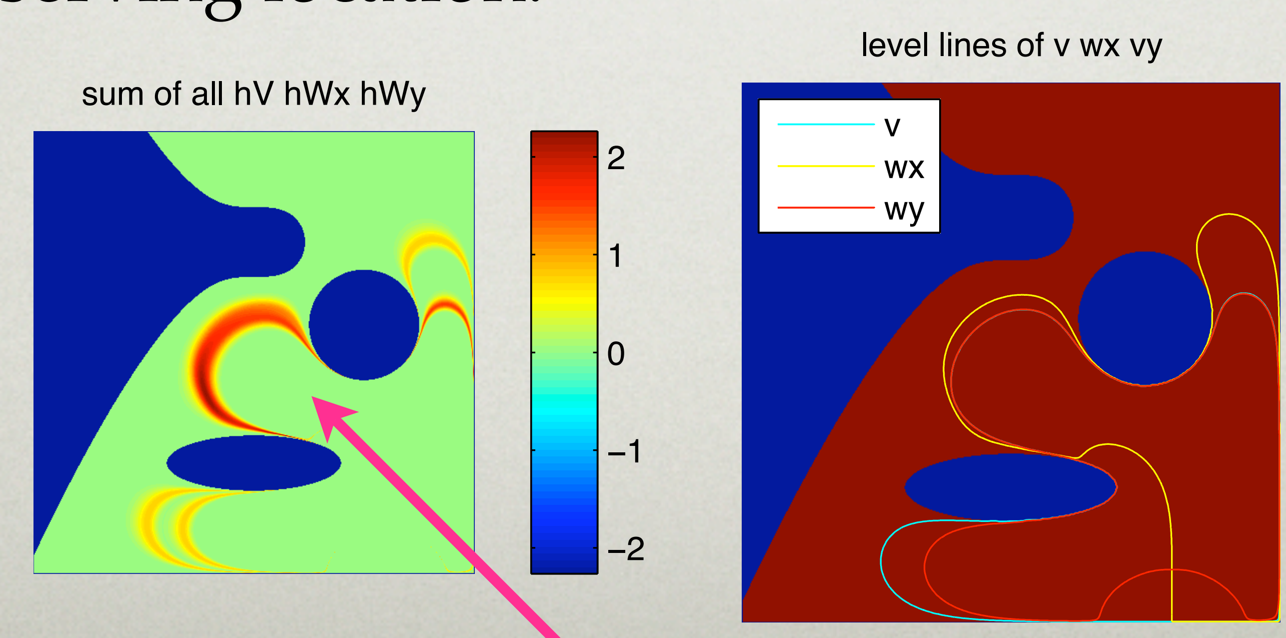
Adjusted “sniffing”: $\tilde{S}_2 \in \{v = I - \tilde{u}_1(O)\}$ $\quad I = \sum u_k(O)$

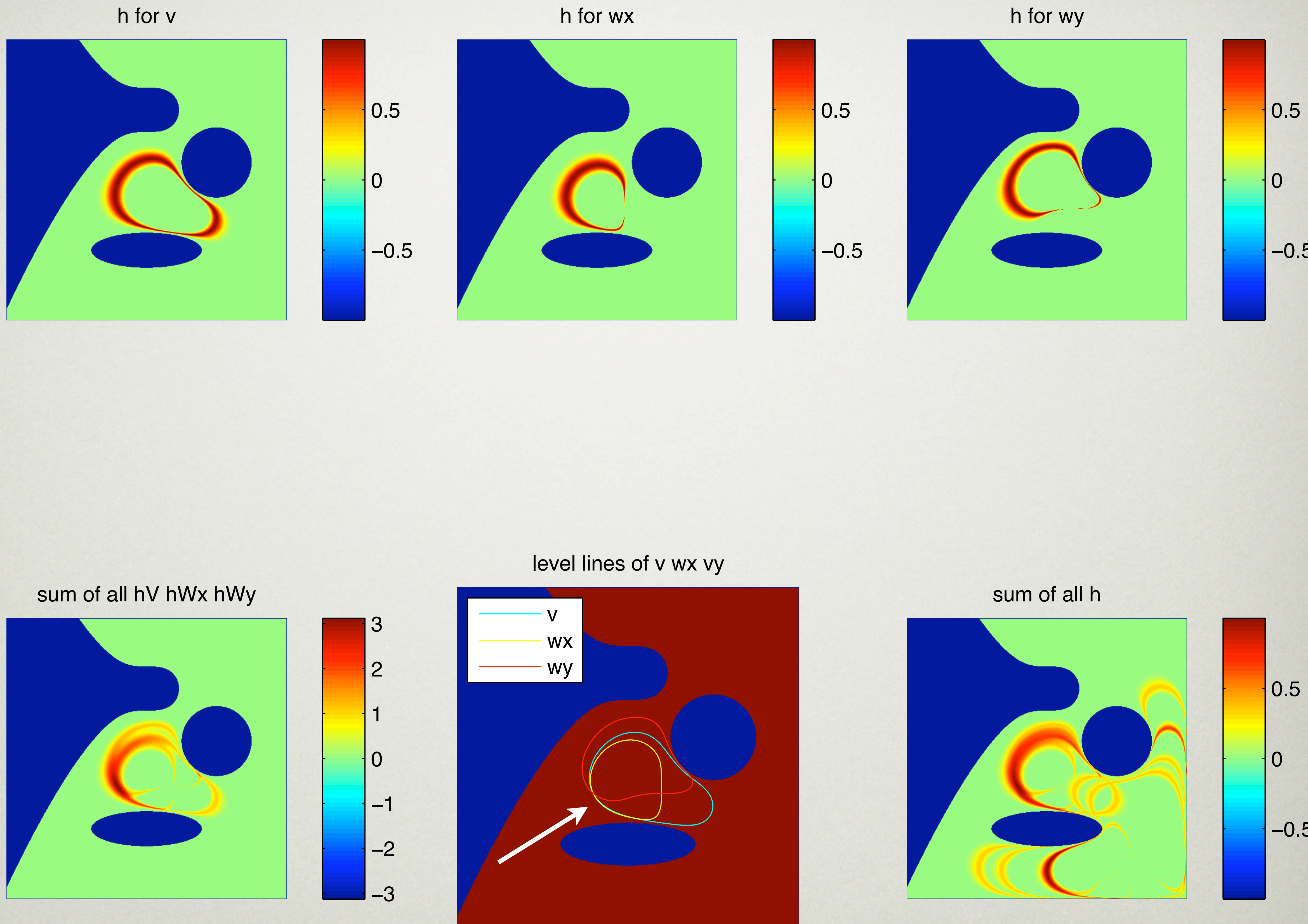


Identified souce: $\tilde{S}_1$ $$-\Delta \tilde{u}_1 = \delta(x - \tilde{S}_1), \quad \tilde{u}_j|_{\partial\Omega} \equiv 0$$

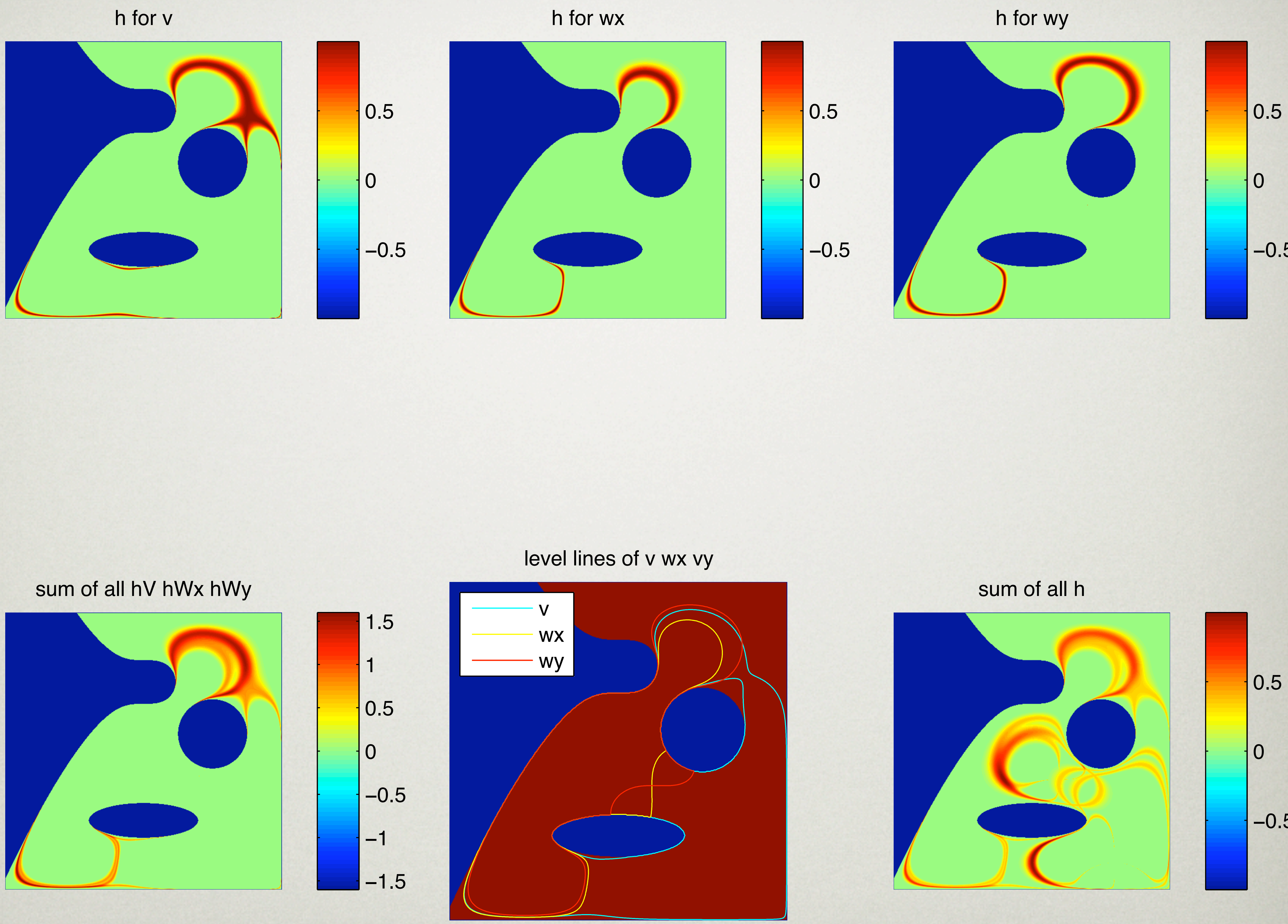
Adjusted "sniffing": $\tilde{S}_2 \in \{v = I - \tilde{u}_1(O)\}$ $$I = \sum u_k(O)$$

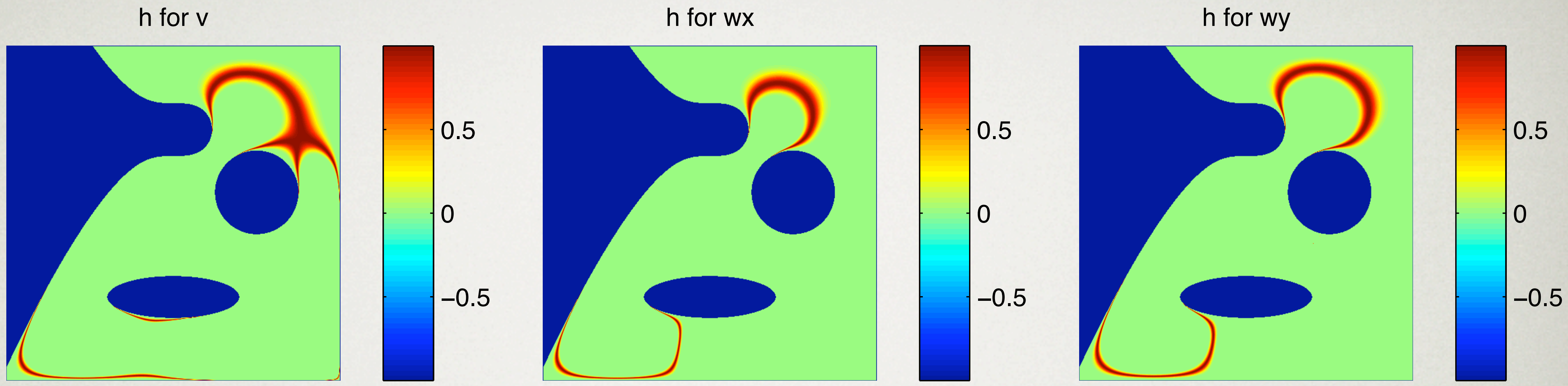
Hint for new observing location.



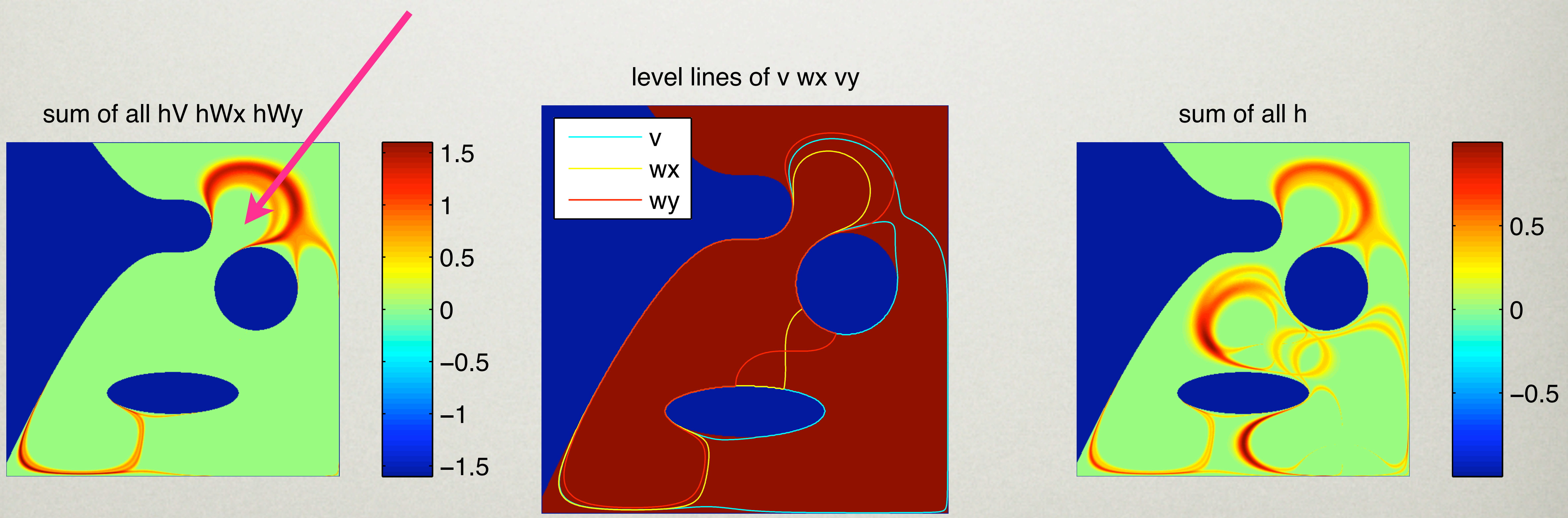


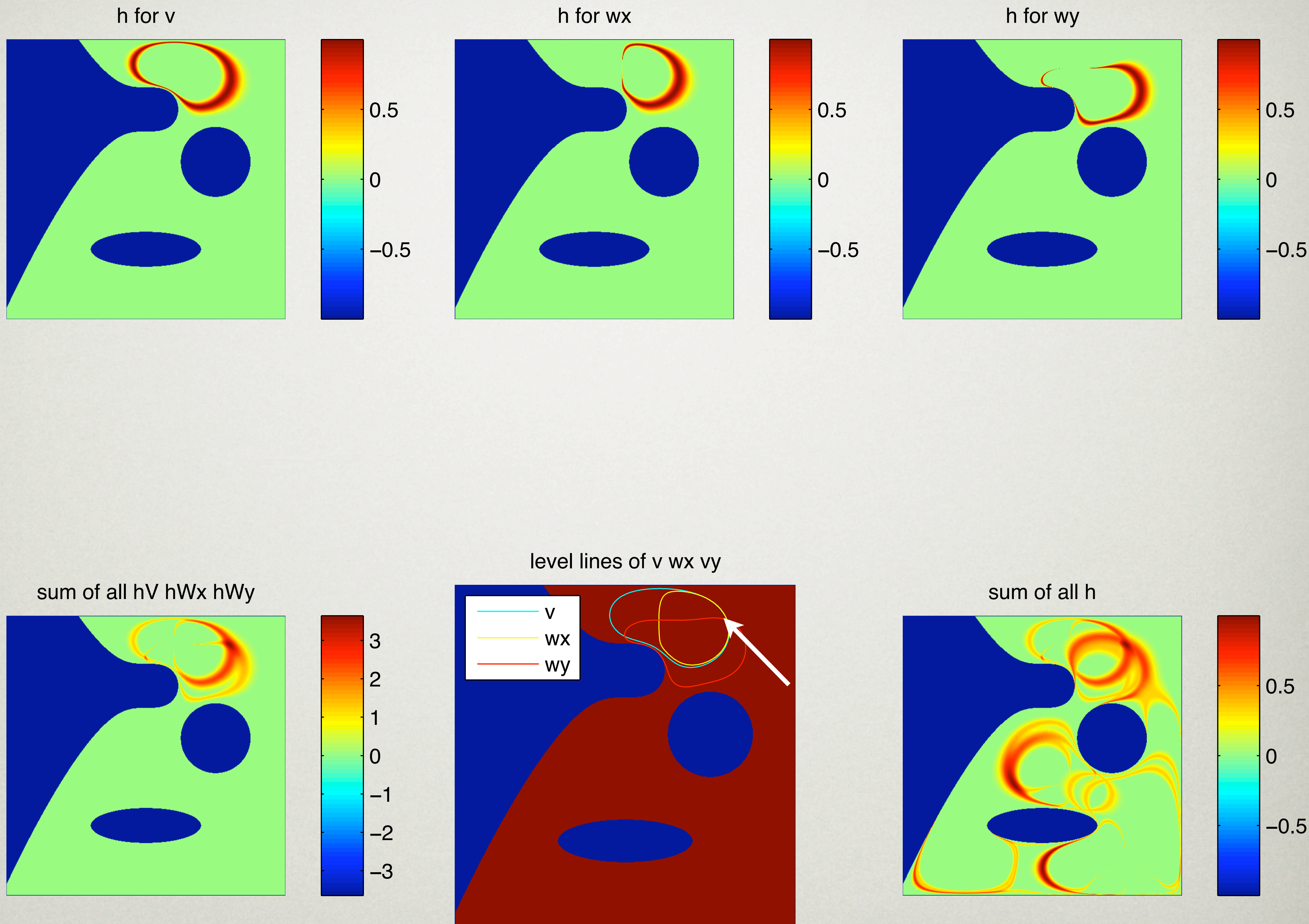
h for v
h for wx
h for wy
0.5
0
−0.5
sum of all hV hWx hWy
3
2
1
0
−1
−2
−3
level lines of v wx vy
v
wx
wy
sum of all h


h for v
0.5
0
−0.5
h for wx
0.5
0
−0.5
h for wy
0.5
0
−0.5
sum of all hV hWx hWy
1.5
1
0.5
0
−0.5
−1
−1.5
level lines of v wx vy
v
wx
wy
sum of all h
0.5
0
−0.5

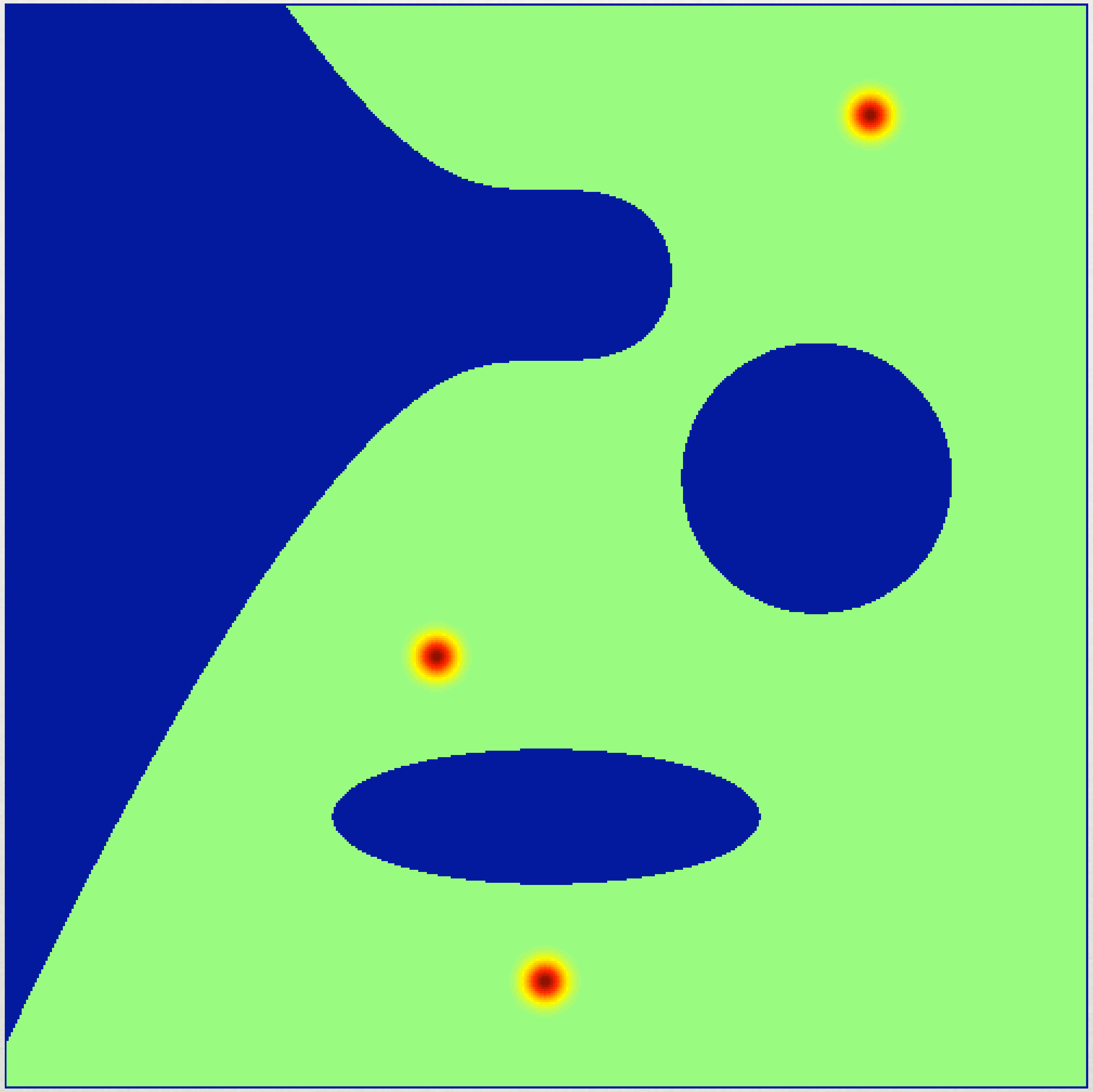


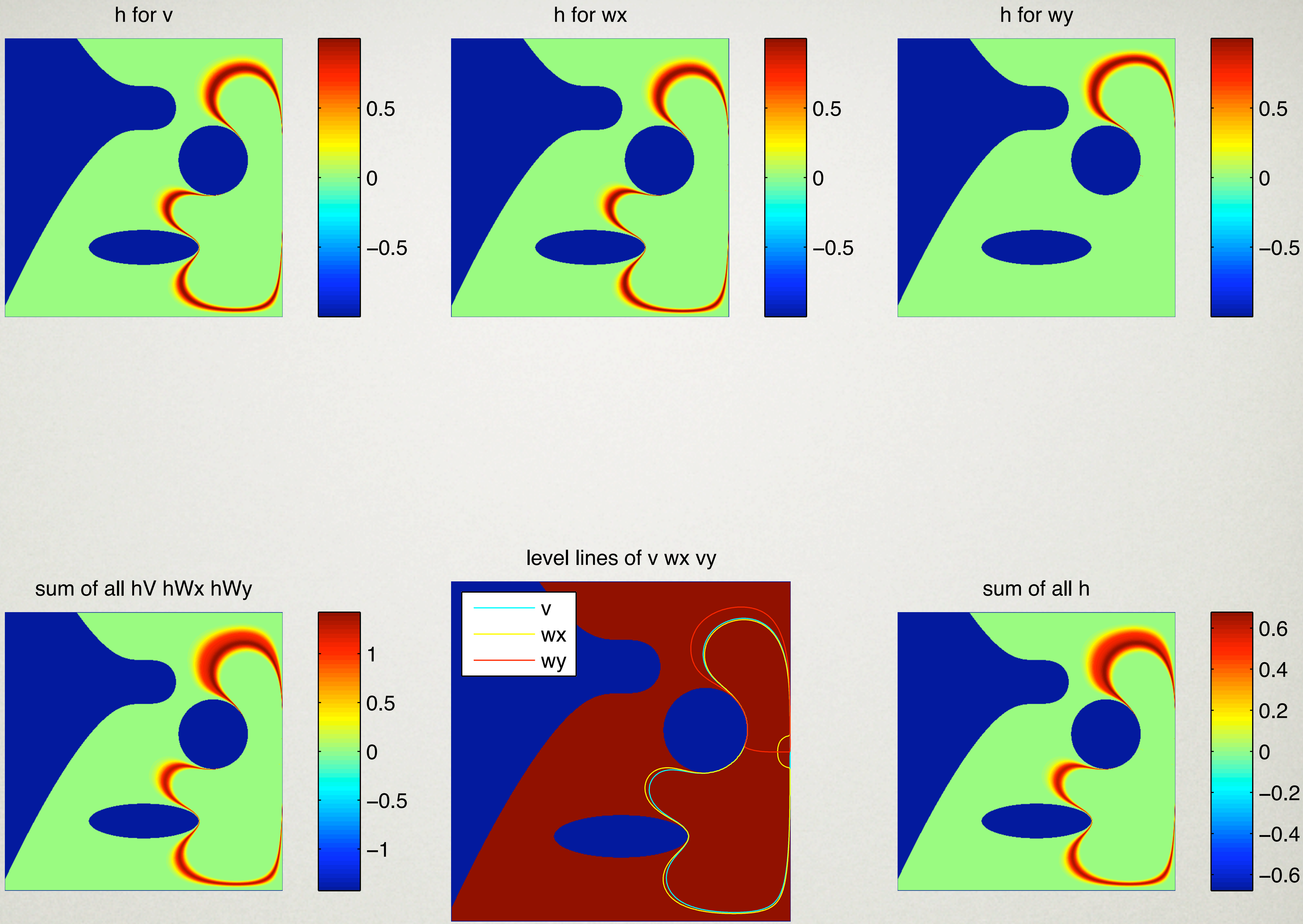
Hint for new observing location.

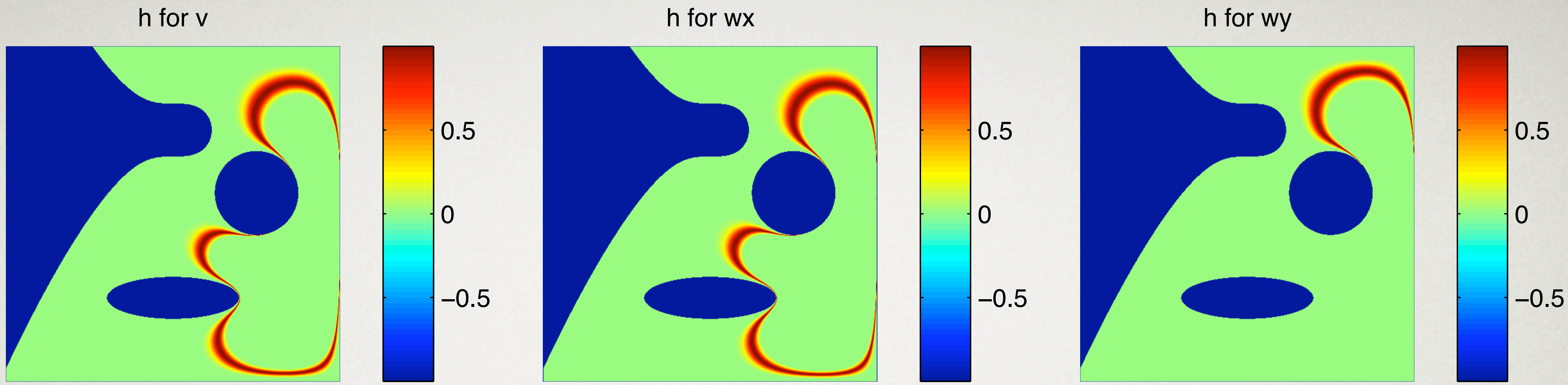



h for v
h for wx
h for wy
0.5
0
−0.5
sum of all hV hWx hWy
3
2
1
0
−1
−2
−3
level lines of v wx vy
v
wx
wy
sum of all h

Source function

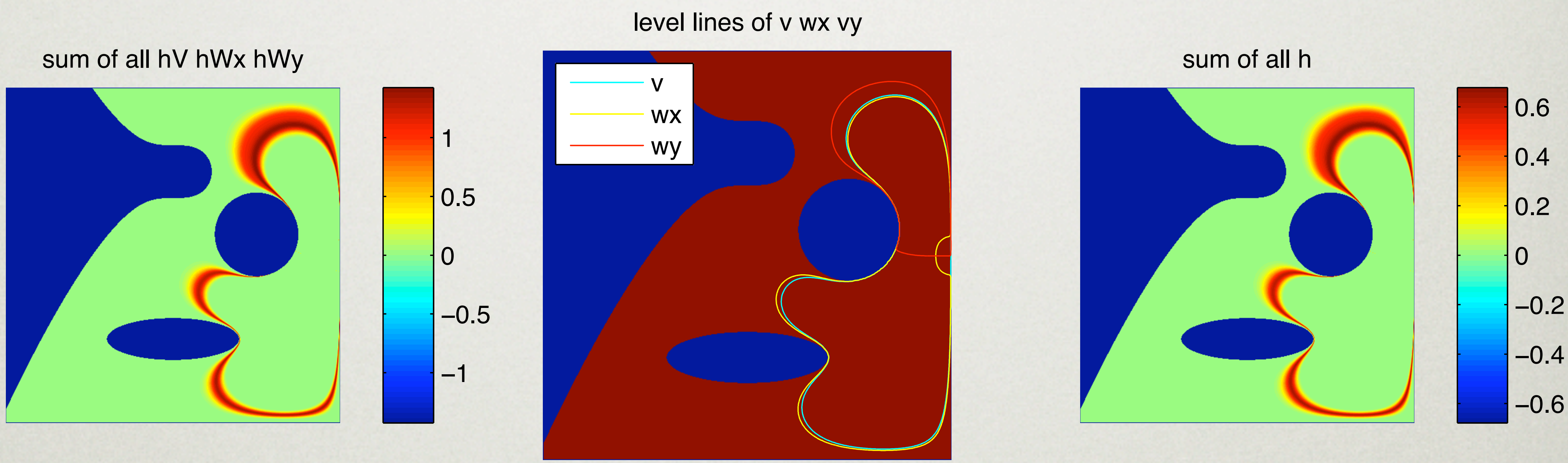


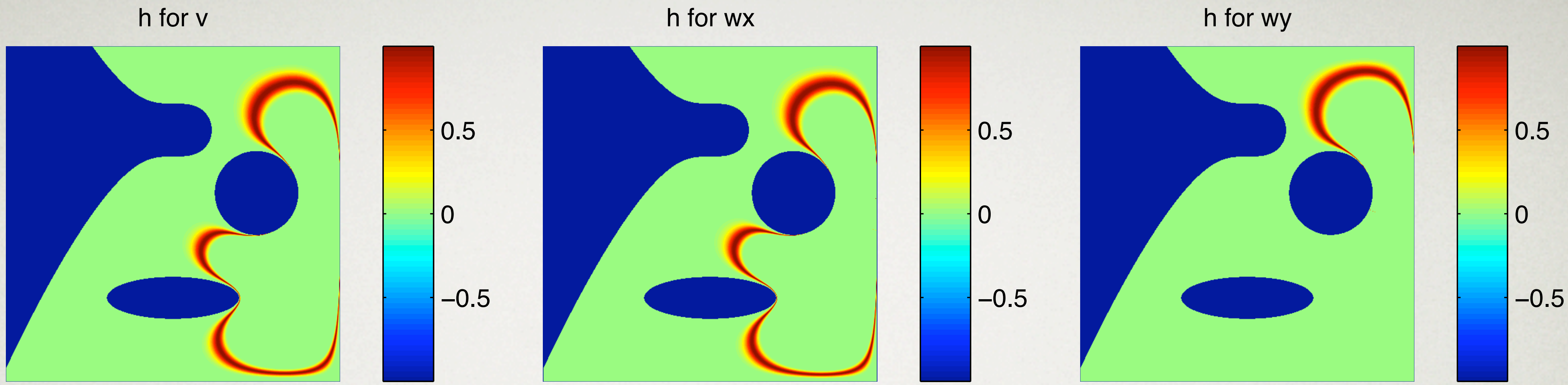

h for v
0.5
0
−0.5
h for wx
0.5
0
−0.5
h for wy
0.5
0
−0.5
sum of all hV hWx hWy
1
0.5
0
−0.5
−1
level lines of v wx vy
v
wx
wy
sum of all h
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



Useless information from three random observing locations.

Effective source locations drastically different from the truth.





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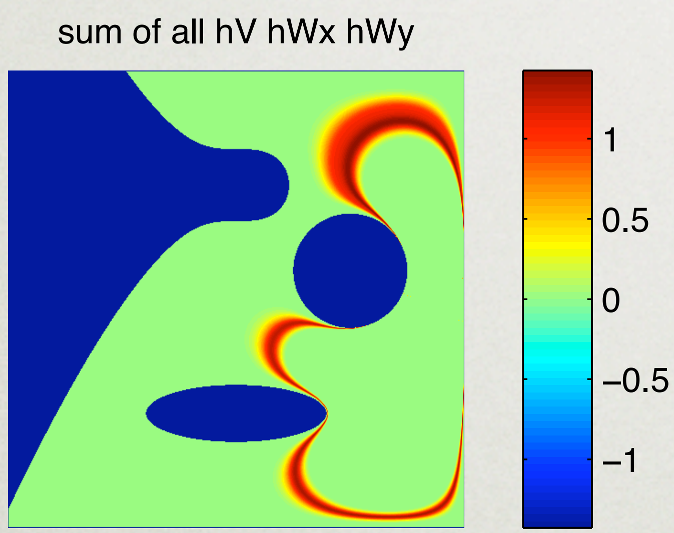
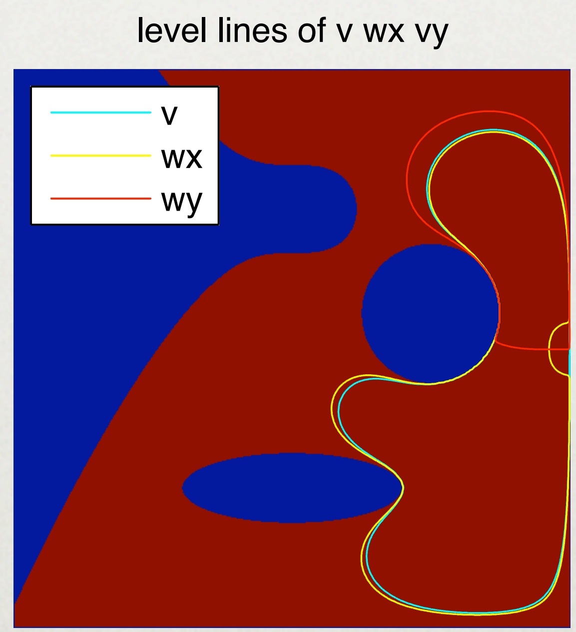
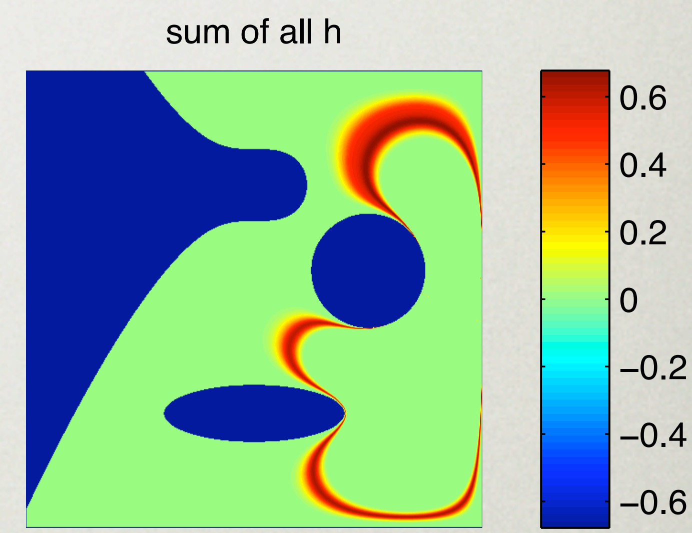
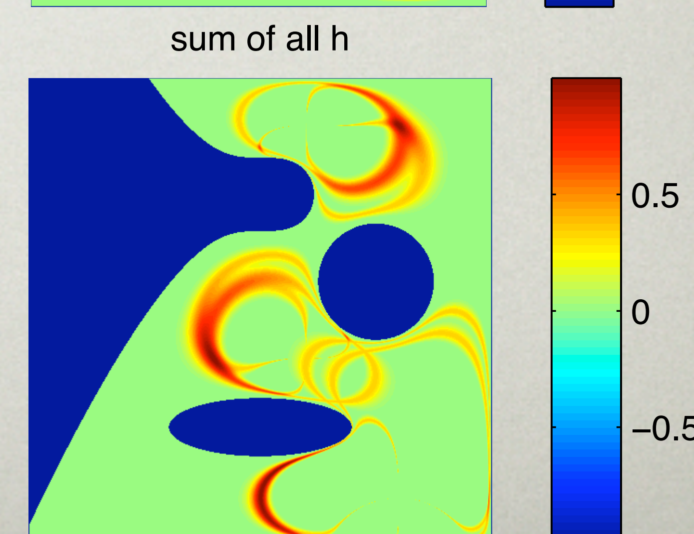






Result by a “smarter” sampling →



Discussion

- Algorithm applicable to other linear problems (self-adjoint or not)
- Stability: bounds on the gradients of u
- Path strategy
- Multiple sources of different (unknown) strengths
- Non-standard inverse problems (fewer sensors, freedom in sensor locations, two modalities)