Chapter 5 - Finance

- **Simple Interest** - interest that is computed on the original principal only

- **Simple Interest Formulas**
  
  \[ I = Prt \]

  Accumulated Amount \( A = P + I = P + Prt = P(1 + rt) \)

  **NOTATION:** \( I \) = interest earned, \( P \) = principal, \( r \) = interest rate (as a decimal), \( t \) = term of the investment in YEARS, \( A \) = accumulated amount

- **The TVM-Solver CANNOT be used for simple interest calculations.**

- **Compound Interest** - earned interest that is periodically added to the principal and thereafter itself earns interest at the same rate.

- The TVM-Solver can be used in problems involving compound interest as follows:

  \( N \) = total number of payments made, usually \( m \times t \).

  \( I_n \) = interest rate in percent form. Don’t convert to decimal form!!

  \( PV \) = present value (principal, or the amount you start with). Entered as negative if invested, positive if borrowed.

  \( PMT \) = payment (amount paid each period). Entered as negative if paying off a loan, positive if receiving money. 0 if computing compound interest.

  \( FV \) = future value (accumulated amount). This will be 0 if paying off a loan.

  \( P/Y \) = number of payments per year (usually the same as \( m \)).

  \( C/Y \) = number of conversions per year (\( n \)).

- At the bottom of the screen, you will see PMT:END BEGIN. If END is highlighted, then the TVM Solver calculates everything with payments being made at the end of the period. For virtually all of the problems we will work in class, END should be highlighted.

- You can solve for any quantity on the TVM-Solver by moving the cursor to that quantity and then pressing ALPHAN followed by ENTER.

- **Effective Rate of Interest** - The effective rate of interest is a way of comparing interest rates. More precisely, the effective rate is the simple interest rate that would produce the same accumulated amount in 1 year as the nominal rate compounded \( n \) times per year.

- The effective rate of interest is typically denoted by \( r_{eff} \) and is also known as the effective annual yield.

- To calculate the effective rate of interest, use the Eff ) function on the calculator. This function can be found under Finance—just arrow down until you see C: Eff.

- The Eff ) function has two parameters, the nominal (or annual) interest rate entered as a percent, and the number of conversion, \( n \), per year. Eff(nominal rate as a percent, \( n \))

- **Annuity** - a sequence of payments made at regular time intervals.

- In this course, we will study annuities with the following properties:
  1. The terms are given by fixed time intervals.
  2. The periodic payments are equal in size.
  3. The payments are made at the end of the payment periods.
  4. The payment periods coincide with the interest conversion periods.
1. Jake deposited $350 into an account paying 3.25% simple interest. How much money is in the account at the end of 4 years? How much interest was earned?

\[ I = Prt = 350 \times 0.0325 \times 4 \]
\[ I = 45.50 \]

\[ A = P + I = 350 + 45.50 = 395.50 \]

2. When Erica graduated from high school, she received $500 from her parents as a gift. She then loaned this money to her brother who repaid her 3 months later with a sum of $510.25. What was the simple interest rate that Erica charged her brother?

\[ I = Prt \]
\[ 10.25 = 500 \times r \times \frac{3}{12} \]
\[ r = 0.082 \]

Simple interest rate was 8.2%
3. Annette wants to take a trip to Europe when she graduates. She will need $4,500 for this trip. How much money should Annette deposit now into an account paying 8%/year compounded quarterly if she expects to graduate in 4 years? How much interest will she earn?

\[
\begin{align*}
N &= 4 \times 4 \\
\text{I}\% &= 8 \\
\text{PV} &= ? \\
\text{PMT} &= 0 \\
\text{FV} &= 4500 \\
\text{P/Y} = \text{C/Y} &= 4
\end{align*}
\]

Interest earned = \[
\frac{4500 - 3278.01}{1} = 1221.99
\]

4. Lynnette, Annette’s twin sister, wants to take that same trip to Europe, but she does not have enough money to open the same type of account as Annette. Instead, she plans to make monthly payments to an account paying 8.25%/year compounded monthly. How much should each payment be so that she has $4,500 at the end of 4 years? How much interest will Lynnette earn?

\[
\begin{align*}
N &= 4 \times 12 \\
\text{I}\% &= 8.25 \\
\text{PV} &= 0 \\
\text{PMT} &= ? \\
\text{FV} &= 4500 \\
\text{P/Y} = \text{C/Y} &= 12
\end{align*}
\]

Lynnette’s total deposit = \[
(79.45)(4 \times 12) = 3813.60
\]

Interest earned = \[
4500 - 3813.60 = 686.40
\]
5. Kira opened an account paying 5.25%/year compounded monthly with $100 and plans to add $50 at the end of each month until she has at least $45,000. How long will it take her to first reach her goal? How much will she actually have in the account when she first reaches her goal?

\[ N = \frac{FV}{PMT} \]  
\[ N = \frac{363,798}{350} \]  
\[ N = 363.798 \]

Kira needs 364 payments to have a FV of at least $45,000.

\[ \frac{364}{12} = 30.333333333333333 \]

\[ 30 \text{ yrs } 4 \text{ months} \]

\[ N = 364 \]  
\[ i = 0.0525 \]  
\[ PV = -100 \]  
\[ PMT = -50 \]  
\[ FV = ? \]  
\[ 364 \]  
\[ \text{ comes back as positive!} \]  
\[ FV = 45,049.64 \]
6. Benjamin is 25 years old and plans to retire in 40 years. When he retires, he would like to receive monthly payments of $3,000 from a retirement account for 15 years.

(a) How much money should Benjamin deposit at the end of each month from now until he retires to achieve this goal if he secures an account that will pay 6.25% per year compounded monthly for the life of the account?

\[
\begin{align*}
\text{Step 1: Find out how much} & \quad \text{Ben will need in the account when he retires.} \\
N = 12 \times 40 & \quad \text{(12 months per year for 40 years)} \\
FV = 30000 & \\
PV = ? & \\
\text{PMT} = ? & \\
\text{I/Y} = 5 & \quad \text{(5% per year)}
\end{align*}
\]

\[
\begin{align*}
FV & = PMT \times \frac{(1 + \frac{I}{Y})^N - 1}{\frac{I}{Y}} \\
30000 & = PMT \times \frac{(1 + \frac{5}{12})^{12 \times 40} - 1}{\frac{5}{12}}
\end{align*}
\]

\[
\begin{align*}
PMT & = \frac{30000 \times \frac{5}{12}}{(1 + \frac{5}{12})^{12 \times 40} - 1} \\
PMT & \approx 114.12
\end{align*}
\]

(b) How much will Benjamin deposit into this account?

\[
\begin{align*}
114.12 \times 12 \times 40 & = \$78,777.60
\end{align*}
\]

(c) How much interest will be earned over the entire life of the account?

\[
\begin{align*}
\text{Total interest earned} & = \text{Interest earned before retirement} + \text{Interest earned during retirement} \\
& = (349,885.70 - 78,777.60) + (3000 \times 150 \times 349,885.70) \\
& = 271,008.04 + 15,176,422.50 \\
& = 15,447,430.54
\end{align*}
\]

Total interest earned can also be found as:

\[
\begin{align*}
\text{Interest earned} & = \text{Amount Ben withdraws during retirement - Total amount Ben deposited into the account} \\
& = 8000 \times 150 - 78,777.60 \\
& = 540,000 - 78,777.60 \\
& = 461,222.40
\end{align*}
\]
7. Miles and Keiko are shopping for a new home. They can afford a down payment of $25,000 and monthly payments of at most $850. Bank A has offered to finance a loan at 8.75%/year compounded monthly for 30 years, whereas Bank B has offered 8.25%/year compounded monthly for 25 years.

(a) What is the most expensive house they can afford to buy? Which bank would they have to use for this house?

**Bank A**

\[ \begin{align*} \text{N} &= 12 \times 30 = 360 \\ \text{I/Y} &= 8.75 \\ \text{PV} &= \text{?} \\ \text{FV} &= 0 \\ \text{PMT} &= -850 \quad \text{paid back as principal} \\ \text{FV} &= 0 \\ \text{I/Y} &= 12 \end{align*} \]

(b) Miles and Keiko ultimately make a down payment of $25,000 on a $110,000 home and finance the balance through Bank B. What monthly payments should they make to pay off the house in 25 years? How much interest did they pay?

\[ \begin{align*} \text{N} &= 25 \times 12 = 300 \\ \text{I/Y} &= 8.25 \\ \text{PV} &= 110,000 - 25,000 = 85,000 \quad \text{and of the loan} \\ \text{FV} &= 0 \\ \text{PMT} &= \text{?} \\ \text{I/Y} &= 12 \quad \text{consolidated as negative} \end{align*} \]

Total amount paid for house:

\[ 85,000 + 670.18 \times 25 \times 12 = \$294,054 \]

Amount paid in interest:

\[ 294,054 - 110,000 = \$184,054 \]

(c) Refering to part (b), create an amortization schedule for the first 4 months of the loan:

<table>
<thead>
<tr>
<th>Period</th>
<th>Interest Owed</th>
<th>Payment</th>
<th>Amount Toward Principal</th>
<th>Outstanding Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>85000</td>
<td>841.42</td>
<td>841.42</td>
<td>85000</td>
</tr>
<tr>
<td>1</td>
<td>841.42</td>
<td>841.42</td>
<td>841.42</td>
<td>85000 - 841.42 = 84158.58</td>
</tr>
<tr>
<td>2</td>
<td>841.42</td>
<td>841.42</td>
<td>841.42</td>
<td>85000 - 841.42 = 84158.58</td>
</tr>
<tr>
<td>3</td>
<td>841.42</td>
<td>841.42</td>
<td>841.42</td>
<td>85000 - 841.42 = 84158.58</td>
</tr>
<tr>
<td>4</td>
<td>841.42</td>
<td>841.42</td>
<td>841.42</td>
<td>85000 - 841.42 = 84158.58</td>
</tr>
</tbody>
</table>

\[ 841.42 \times 0.0825/2 = 843.79 \]

Title: May 1 - 6:03 PM (8 of 14)
8. What is the present value of a sum of $5,000 due in 6 years at an interest rate of 6.75% per year compounded continuously?

\[ A = Pe^{rt} \]

\[ 5000 = Pe^{(0.0675)(6)} \]

\[ P = \frac{5000}{e^{(0.0675)(6)}} \]

\[ P = $3,334.88 \]

Dirty trick:
\[ N = (1.099)^6 \]
\[ 1.099 = 6.75 \]
\[ PV = ? \rightarrow $3,334.88 \]
\[ PMT = 0 \]
\[ FV = 5000 \]
\[ P/Y = C/Y = 1 \]

Title: May 1 - 6:03 PM (9 of 14)
9. Julian opened an account with $8,000 and after 7 years, it had grown to $10,000.

(a) What was the annual interest rate if interest was compounded weekly?

\[ N = 7 \times 52 \]
\[ I_{\%} = \frac{FV - PV}{PV} \times 100 \]
\[ PV = -8000 \]
\[ FV = 10000 \]
\[ P = \frac{FV}{N 	imes m} = 52 \]

(b) If the annual interest rate found in part (a) was instead a simple interest rate, how long would it take for Julian’s $8,000 to grow to $10,000?

\[ I = Prt \]
\[ 2000 = 8000 \times \left(0.03187\right) t \]
\[ t = 7.8402 \text{ years} \]

about 7 years and 10 months.

10. If Bank A has a savings account paying 8% per year compounded semiannually and Bank B offers 7.9% per year compounded monthly, which is the better offer?

We need to find the effective rate (effective annual yield) for each.

**Bank A:** \[ \text{Eff}(8,2) = 8.16\% \]

**Bank B:** \[ \text{Eff}(7.9,12) = 8.1924\% \]

Bank B is better.
11. Juanita decided to purchase a flat-screen HDTV. She makes a down payment of $250 and secures financing for the balance of the purchase price at a rate of 12%/year compounded monthly. Under the terms of the finance agreement, she is required to make monthly payments of $125 for 30 months.

(a) What was the cash price of the TV?

Need to find the original amount of the loan.

\[
\begin{align*}
N &= 30 \\
\text{FV} &= 12 \\
\text{PV} &= \text{amount of loan} = 3225.96 \\
\text{PMT} &= -125 \\
\text{FV} &= 0 \\
\text{PMT} &= \text{N} = 12
\end{align*}
\]

Total cost of TV = \[
3225.96 + 250 = \boxed{3475.96}
\]

(b) How much interest did Juanita pay?

\[
\text{Total amount paid for TV} = 250 + 125 \times 30 = 4000
\]

Interest paid = \[
4000 - 3475.96 = \boxed{524.04}
\]
12. Deanna owes $1,000 on a credit card that has an interest rate of 22.5%/year compounded monthly. If she pays the minimum payment of $20 each month,

(a) how much of her first payment goes toward interest?

\[
\text{Owes } \$1000 \text{ total. After 1 month, she will owe } 1000 \times (0.225/12) = \$18.75
\]

(b) how long will it take her to pay off the card? (Assume no additional charges are made.)

\[
\begin{align*}
N &= 149.2534 & \text{(Make sure to round to appropriate number of significant figures.)} \\
I_{10} &= 22.5 \\
P &= 1000 \\
PMT &= -20 \\
FV &= 0 \\
p/Y &= c/Y = 12
\end{align*}
\]

\[
150/12 = 12.5 \text{ years}
\]

\[
12 \text{ years + } 6 \text{ months to pay off the card}
\]
13. The Gardners purchased a vacation home 15 years ago. At the time of the purchase, they were able to make a down payment of 20% of the purchase price and then secured a loan of $105,000 to finance the remaining amount. The loan was to be amortized with monthly payments over 30 years at an interest rate of 6.75% per year compounded monthly.

(a) What is the current outstanding principal on the loan?

\[ \text{Step 1: Calculate the monthly payment.} \]

\[ N = 30 \times 12 \]
\[ \text{r}_0 = 0.75 \]
\[ PV = 105,000 \]
\[ PMT = ? \rightarrow $181.03 \]
\[ FV = 0 \quad \text{(same as previous)} \]
\[ P/Y=C/Y = 12 \]

\[ \text{Step 2: Calculate the amount paid on the loan.} \]

\[ N = 15 \times 12 \]
\[ \text{r}_0 = 0.75 \]
\[ PV = 105,000 \]
\[ PMT = -681.03 \]
\[ FV = \rightarrow 76,959.57 \]
\[ P/Y=C/Y = 12 \]

(b) How much equity do the Gardners have in their vacation home?

Equity = value of the house - what you still owe

To find the value of the house:

We made a 80% down payment and got a loan for $105,000, so $105,000 is 80% of the total cost of the house:

\[ 105,000 = 0.8 \text{(cost of house)} \]
\[ 131,250 = \text{cost of house} \]

Equity = \[ 131,250 - 76,959.57 \]
\[ = \$54,290.43 \]
35 different stools total

11 had backs (B)
20 were padded (P)
26 padded or have backs

What is the probability that a padded stool is backless?

\[ P(B^c | P) = \frac{P(B^c \cap P)}{P(P)} = \frac{\frac{15}{35}}{\frac{20}{35}} = \frac{15}{20} = \frac{3}{4} \]

\[ n(B) = 11, \quad n(P) = 20, \quad n(P \cup B) = 26 \]

\[ n(P \cup B) = n(P) + n(B) - n(P \cap B) \]

\[ 26 = 20 + 11 - n(P \cap B) \]

\[ 5 = n(P \cap B) \]