MATH 609-601
Homework #9
Multi-step Methods and Their Stability

Solve any set of problems for 100 points.

(1) (20 pts) Consider the Cauchy problem $y' = f(t, y), y(t_0) = y_0$ and its approximation by Adams-Moulton multi-step method. Derive the corresponding formulas for the implicit (closed) multi-step method of order four that uses integration over the interval (t_{i-2}, t_{i+1}). Write down the expression for the local truncation error.

(2) (20 pts) Derive an explicit (open) multi-step method of order four (Adams-Bashforth four step method) that uses integration in the interval (t_i, t_{i+1}). Write down the expression for the local truncation error.

(3) Check the consistency and derive an expression for the local truncation error of the following Adams type formulas (here $\eta_i \approx x(t_i)$):
(a) (10 pts) $\eta_{i+1} = \eta_{i-2} + 1.5h(f_i + f_{i-1})$;
(b) (10 pts) $\eta_{i+1} = \eta_{i-1} + 1/3h(7f_i - 2f_{i-1} + f_{i-2})$;
(c) (10 pts) $\eta_{i+1} = \eta_{i-1} + 1/3h(f_{i+1} + 4f_i + f_{i-1})$;
Note that here $f_i = f(t_i, \eta_i)$.

(4) Solve the following difference equations and check for the stability condition:
(a) (10 pts) $\eta_{i+1} - 2\eta_i + \eta_{i-1} = 0, \eta_0 = 1, \eta_1 = 0$;
(b) (10 pts) $\eta_{i+1} - \eta_i - \eta_{i-1} = 0, \eta_0 = 0, \eta_1 = 1$;
(c) (10 pts) $6\eta_{i+1} - 5\eta_i + \eta_{i-1} = 0$, (find the general solution);
(d) (10 pts) $2\eta_{i+1} - 5\eta_i + 2\eta_{i-1} = 0$, (find the general solution).

(5) (20 pts) Consider the initial value problem $y' = f(t, y), y(t_0) = y_0$. Show that the multi-step method

$$\eta_n - \frac{4}{3}\eta_{n-1} + \frac{1}{3}\eta_{n-2} = \frac{2}{3}hf_n, \; n = 2, 3, \ldots$$

and $\eta_0 = y_0$ and η_1 properly chosen, is:
(a) consistent;
(b) stable;
(c) A-stable (this one is very difficult !!!).

(6) (10 pts) Consider the initial value problem $y' = f(t, y), y(t_0) = y_0$. Show that the mid-point method

$$\eta_n - \eta_{n-2} = 2hf_n, \; n = 2, 3, \ldots$$

and $\eta_0 = y_0$ and η_1 properly chosen, is NOT A-stable.