

MATH 609 Numerical Analysis
Programming assignment #2
Gauss-Seidel and SOR iterative methods

1 Problem Formulation

Write a program for solving the system $Ax = b$ (generated by Problems 2 and 3 of Programming assignment # 1) by applying: 1) Gauss-Seidel, 2) SOR (with optimal choice of the parameter ω whenever possible), and 3) SSOR. For Problem 2 take $n = 25, 50, 100$ while for Problem 3 take $n = 8, 16, 32$. Use the standard for SOR splitting of A , namely, $A = D - L - U$, where D is the diagonal of A , L is the lower triangular and U the upper triangular matrices.

2 Specifications

1. For stopping the iterations use any of the conditions: (1) $\|x^{(m+1)} - x^{(m)}\|_2 / \|x^{(0)}\|_2 < TOL$, (2) $\|r^{(m)}\|_2 / \|r^{(0)}\|_2 < TOL$, where $r^{(m)}$ is the residual of the m -th iterate, or any other criterion you consider appropriate. Set $TOL = 10^{-6}$ or $TOL = 10^{-12}$ if using double precision.
2. Show that if G_{SSOR} is written in the form $G_{SSOR} = I - BA$ and A is symmetric and positive definite then the matrix B is symmetric, i.e. $B = B^T$.
3. In all problems take the following r.h.s. $b = h^2(1, 1, \dots, 1)^t$.
4. The report should be in the specified latex-format (or MS doc format) and must contain tables with the number of iterations for all cases (plots are welcome as well). Compare the number of iteration for these three implementations. *Remark.* If you are getting to big differences in the iteration count then there is something wrong either with your implementation or with the estimates of the parameters a and b , which are related to the estimates of the maximum and minimum eigenvalues of A .
5. Since the solutions x_i and $x_{i,j}$ of these systems represent approximations of the solutions $u(x_i)$ and $u(ih, jh)$ of some boundary value problems, it makes sense to plot them as functions in one and two variables, respectively. Present graphs only for the finest grids.

3 Penalties

There will be penalties for delaying the programming assignment - for each day 5 points (out of 100).