

MATH 609 Numerical Analysis
Programming assignment #3
Chebyshev acceleration and CG iterative methods

1. PROBLEM FORMULATION

Write a program for solving the system $Ax = b$ (generated by Problems 2 and 3 of the Programming assignment 1) by Chebyshev acceleration iterative method. In all of the runs use a zero initial iterate. For Problem 2 take $n = 25, 50, 100$. For Problem 3 take $n = 8, 16, 32$. You can implement the method either a two-term variable parameter (Richardson-like) or as three term relation method from your book (pp. 239 – 244).

Also implement (or use a ready made subroutine supplied by your TA) Conjugate Gradient Method and solve the same problems as above. Make comparison of the results in terms of iteration count.

2. SPECIFICATIONS

- (1) Let $r^{(m)} = b - Ax^{(m)}$ be the residual of the m -iterate. Stop the Chebyshev iteration process when $\|r^{(m)}\|/\|r^{(0)}\| \leq TOL$, where $\|x\|^2 = \sum |x_i|^2$ for any vector $x \in \mathcal{R}^n$ and set $TOL = 10^{-6}$. For the CG method use the A -norm for the residual. You may experiment with $TOL = 10^{-12}$ if using double precision.
- (2) First, in both problems choose $b = h^2(1, 1, \dots, 1)^t$.
- (3) For both Problem 2 and Problem 3 of Programming assignment 1 apply Chebyshev acceleration methods using two-term and three-term recurrence methods discussed in class. In the case of two-term recurrence you may use a fixed number of parameters, say 64, and repeat them in a cyclic manner, or determine the number of the parameters in advance.
- (4) The report should be in the specified latex-format and should contain tables with the number of iterations for all cases. In order to have a chance to compare the iteration count with the results of Jacobi and SOR, simply add two new columns to the previous tables.
- (5) Since the solutions x_i and $x_{i,j}$ of these systems represent approximations of the solutions $u(x_i)$ and $u(ih, jh)$ of some boundary value problems, it makes sense to plot them as functions in one and two variables, respectively. Present graphs only for the finest grids.

3. PENALTIES

The penalty for delay is 5 points per day (out of 100).