MATH 609 Numerical Analysis
Programming assignment #4
Cubic Spline Interpolation

1. Problem Formulation

Write a program for cubic spline interpolation in the framework discussed in class and in the textbook of Kincaid and Cheney *Numerical Analysis*, chapter 6, section 6.4.

2. Specifications

(1) The program should be written to input from a data file, the number of knots n, the interpolation points t_i, the functional values y_i, $i = 0, \ldots, n$, and the boundary conditions.
(2) The solution of the system of linear equations for the unknown parameters z_i (see your notes or the textbook) should be done in a separate subroutine. You can use any of the solvers developed in the previous programming assignments.
(3) The report should be in the required form and should contain short description of the method and the problems and plots of the obtained spline curves.

3. Computational examples

Solve the following linear systems:

(1) A car traveling along a straight road is clocked at a number of points. The data from the observations is given in the following table where the time is in seconds, and the distance in feet:

Time	0	3	5	8	13
Distance	0	225	385	623	933

(2) Example 1. Use cubic spline interpolation with free ends to predict the position of the car and its speed when $t = 10$ sec.

Example 2. Use cubic spline interpolation with fixed ends to predict the position of the car and its speed when $t = 10$ sec. In this case use the fact that the speed at the beginning and the end of the race is 75 and 72 feet per second, respectively.

Example 3. Use cubic spline interpolation with free ends to find the population of USA in 1965, 1975 and 1985 from the data:

Year	1930	1940	1950	1960	1970	1980	1990
Population (*in K*)	123,203	131,669	150,697	179,323	203,212	226,505	249,643

(3) Now consider the the curve C on the (x, y)-plane given by $C: \ (x(t), y(t)), \ a \le t \le b$, where the values of $x(t)$ and $y(t)$ are given at the knots $a = t_0 < t_1 < \ldots < t_n = b$. Approximate this curve by $(S_x(t), S_y(t))$, where S_x and S_y are natural cubic spline interpolants of $x(t)$ and $y(t)$, respectively, at the given knots.

Example 1. Consider the case $x(t) = cost$, $y(t) = sint$, on $(0, \pi)$. Take $n = 11$ and $n = 21$. Produce a plot.

Example 2. Problem 7 of Computer Problems Set 6.4, page 391 of your textbook. Produce a plot.