**MATH 609-601 Numerical Analysis**
**Programming assignment #4**
**Runge-Kutta Method with Automatic Step-Size Control**

Write a program for solving Cauchy problem for a single differential equation of first order applying one of the methods that automatically controls the error less than $\epsilon$ by selecting appropriate step-size.

**Specifications**

1. Run the program for two values of $\epsilon$: $10^{-2}$ and $10^{-4}$.
2. Report the number of steps and the obtained solution at the end point. Plot the obtained approximate solution in the whole interval.

**Computational examples** - solve the following problems:

1. In a circuit impressed, voltage $E$, and resistance $R$, inductance $L$, capacitance $C$ in parallel, the current satisfies the differential equation
$$i'(t) = CE''(t) + \frac{1}{R}E'(t) + \frac{1}{L}E.$$
Suppose $C = 0.3$ farad, $R = 1.4$ ohms, $L = 1.7$ henries, and the voltage is given by $E(t) = e^{-0.06\pi t} sin(2t - \pi)$. If $i(0) = 0$, find the current $i(t)$ for $0 < t < 5$.

2. A liquid of low viscosity, such as water, flows from an inverted conical tank with circular orifice at the rate
$$x'(t) = -0.6\pi r^2 \sqrt{2g}\frac{\sqrt{x}}{A(x)},$$
where $r$ is the radius of the orifice, $x$ is the height of the liquid level from the vertex of the cone, and $A(x)$ is the area of the cross section of the tank $x$ units above the orifice. Suppose $r = 0.1$ feet, $g = 32 feet/sec^2$, and the tank has an initial water level of 8 feet and initial volume of $512\pi/3$ cubic feet. Find the time when the tank is emptied.

3. The irreversible chemical reaction in which two molecules of solid potassium dichomate ($K_2Cr_2O_7$), two molecules of water ($H_2O$), and three atoms of solid sulfur ($S$) combine to yield three molecules of the gas sulfur dioxide ($SO_2$), four molecules of solid potassium hydroxide ($KOH$), and two molecules of solid chrome oxide ($Cr_2O_3$) can be represented symbolically by the stoichiometric equation:
$2K_2Cr_2O_7 + 2H_2O + 3S - - - -4KOH + 2Cr_2O_3 + 3SO_2$. If $n_1$ molecules of $K_2Cr_2O_7$, $n_2$ molecules of $H_2O$ and $n_3$ molecules of $S$ are originally available, the following differential equation describes the amount $x(t)$ of $KOH$ after time $t$:
$$x'(t) = k(n_1 - 0.5x)^2(n_2 - 0.5x)^2(n_3 - 0.75x)^3,$$
where $k$ is the velocity constant of the reaction. If $k = 6.22 * 10^{-19}, n_1 = n_2 = 1000, n_3 = 1500$, how many units of potassium hydroxide will have been formed after two seconds ?