MATH 610-600 Numerical Methods for PDEs
Programming assignment #2 & 3
FEM for Two-Point Boundary Value Problems Using Elements of Arbitrary Order

Write a program for solving two-point boundary value problems for second order ordinary differential equations by Ritz-Galerkin method using piece-wise polynomilas of degree $7 \geq p > 1$. Submit a report with graphs of the results, table with the error in discrete L^2- and maximum-norms, and comments (if you have).

Specifications

(1) Use double precision. For solving the corresponding system of linear equations use the program from LAPACK that you discussed on your lab.
(2) Use 10, 20 and 40 finite elements for $p = 2, 4, 7$. Plot the solution. Plot the error.
(3) In a table give the error in L^2, H^1, and maximum-norms.

Computational examples – solve the following problems:

(1) The deflection of a uniformly loaded, long rectangular plate under axial tension force and fixed ends, for small deflections, is governed by the second order differential equation. Let S represent the axial force and q the intensity of the uniform load. The deflection W along the elemental length is given by:

$$W''(x) - \frac{S}{D}W(x) = -\frac{qx}{2D}(l-x), \; 0 < x < l, \; W(0) = W(l) = 0, \tag{1}$$

where l is the length of the plate, and D is the flexural rigidity of the plate. Let $q = 200\ lb/in^2$, $S = 100\ lb/in$, $D = 8.8\ 10^7\ lb\ in$, and $l = 50\ in$. The exact solution is given by: $a = \frac{Sl^2}{D}$, $b = \frac{ql^4}{2D}$, $t = x/l$ and

$$W(t) = \frac{b}{a}\left(-t^2 + t - \frac{2}{a} + \frac{2}{asinh(\sqrt{a})}[sinh(\sqrt{a}t) + sinh(\sqrt{a}(1-t))]\right).$$

(2) Consider the problem (1) when the r.h.s. $\frac{qx}{2D}(l-x)$ is replaced by the constant $\frac{q}{2D}$. The exact solution is

$$W(t) = \frac{Q}{a}\left(1 - \frac{1}{sinh\sqrt{a}}\left(sinh(\sqrt{a}t) + sinh(\sqrt{a}(1-t))\right)\right), \quad Q = \frac{ql^2}{2D}.$$

(3) Consider the differential equation (1) when the right end of the plate is elasticly supported, i.e. instead of the boundary condition $W(l) = 0$ now we have $W'(l) + \frac{\beta}{D}W(l) = 0$, where β is the characteristic of the elastic support. Solve the problem in the following two cases: (a) $\beta = 0$, i.e. the end is free; (b) $\beta = 2\ 10^{10}\ lb$ (you can try the case of very stiff elastic support, namely, $\beta = 2\ 10^{13}\ lb$). The exact solution for the case of a plate with free end (i.e. $\beta = 0$) is:

$$W(t) = \frac{b}{a}\left(-t^2 + t - \frac{2}{a} + \frac{1}{acosh(\sqrt{a})}[\sqrt{a}sinh(\sqrt{a}t) + 2cosh(\sqrt{a}(1-t))]\right).$$

Compare the solution of the case $\beta = 2\ 10^{10}\ lb$ (or $\beta = 2\ 10^{13}\ lb$) with the solution of problem (1).

There will be a penalty for delaying the programming assignment - 5 points per day (out of 100).