

**Final Exam, Math 151, Section 510–512**

**FORM A**

**Instructions.** Work all problems in any order by any method. In the first 14 problems your grade will be determined exclusively by whether you select the correct answer. Circle your answer on this page and also record it on the scantron. In the remaining 6 problems you may receive partial credit for an incomplete answer, so please show enough work. This is a timed test; although we believe you can easily finish in 90 minutes, the time limit is 120 minutes, and no late papers can be accepted.

**Suggestion:** before you begin work, read over the entire test.

Please P R I N T:

NAME:

SECTION:

STUDENT ID:

**Part I.** Multiple Choice, No Calculators (4 points each)

1. At what value of  $x$  does the function  $\frac{(x+1)^2}{(x^2-1)}$  have a removable discontinuity?  
a) -3   b) 3   c) 2   d) -1   e) 1
  
2. Find the equation of the line normal to the curve defined by the equation  $x^3y^4 - 5 = x^3 - x^2 + y$  at the point  $(2,-1)$ .  
a)  $33x + 4y - 62 = 0$    b)  $33x - 4y - 62 = 0$    c)  $33x + 4y + 62 = 0$   
d)  $33x - 4y + 62 = 0$    e)  $4x - 33y - 41 = 0$
  
3.  $\lim_{x \rightarrow \infty} e^{\ln 2x} \sin \frac{1}{x}$   
a)  $-\infty$    b)  $\frac{-1}{2}$    c) 2   d) -2   e) 0
  
4. Compute the derivative of the function  $g(x) = (x + \sin x)^2$ .  
a)  $g'(x) = 2(x + \sin x)(1 + \cos x)$    b)  $g'(x) = 2(1 + \cos x)$    c)  $g'(x) = \frac{1}{3}(x + \sin x)^3$   
d)  $g'(x) = x^2 + 2x \sin x + \sin^2 x$    e)  $g'(x) = 2x + 2 \sin x + 2 \sin x \cos x$
  
5. Tell which of the statements below are true:  
1) If  $f'(c) = 0$ , then  $f(x)$  has a maximum or minimum value at  $x = c$ .  
2) If  $f'(x) = g'(x)$  for all  $x$  in an interval  $I$ , then  $f(x) = g(x)$  on  $I$ .  
3) If  $f(x)$  is differentiable on the open interval  $(a, b)$ , and  $c$  is a point of local maximum for  $f$  in  $(a, b)$ , then  $f'(c) = 0$ .  
a) 1   b) 2   c) 3   d) 1, 2   e) 1, 3
  
6. Let  $f(x) = \int_x^{10} t^3 dt$ . Find the value of  $f'(2)$ .  
a) 8   b) 4   c) -24   d) -4   e) -8

Problems 7–9 deal with the graph of the function

$$f(x) = \frac{x^3}{3} - \frac{3}{2}x^2 + 2x + 1$$

7. Find the interval(s) on which the function  $f$  is decreasing.

- a)  $x < 0$    b)  $x < \frac{3}{2}$    c)  $x > \frac{3}{2}$   
d)  $\{x < 1\} \cup \{x > 2\}$    e)  $1 < x < 2$

8. Find the interval(s) on which the graph of the function  $f$  is concave up.

- a)  $x > 0$    b)  $x < \frac{3}{2}$    c)  $x > \frac{3}{2}$   
d)  $\{x < 1\} \cup \{x > 2\}$    e)  $1 < x < 2$

9. Find the  $x$ -coordinate(s) of all local maxima and minima.

- a)  $x = 1$    b)  $x = 1$  and  $x = 2$    c)  $x = 2$    d)  $x = \frac{3}{2}$    e)  $x = 0$

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10. Find the most general antiderivative of  $\sec^2 x + \cos x$ .

- a)  $\frac{1}{3} \sec^3 x + \sin x + C$    b)  $\tan^2 x - \sin x + C$    c)  $\tan x + \sin x + C$   
d)  $\tan x - \sin x + C$    e)  $\tan^2 x + \sin x + C$

11. Find the area of the region bounded by the parabola  $x = y^2$  and the line  $x - 2y = 3$

- a)  $\frac{29}{3}$    b)  $\frac{32}{3}$    c)  $\frac{35}{3}$    d)  $\frac{38}{3}$    e)  $\frac{41}{3}$

12. Suppose  $f$  is a differentiable function such that  $f(2) = 5$ ,  $f(5) = 2$ ,  $f'(2) = -\frac{1}{2}$  and  $f'(5) = -5$ . Let  $g$  be the inverse function for  $f$ . Let  $L$  be the line tangent to  $y = g(x)$  at  $x = 5$ . Where does this tangent line intersect the  $x$ -axis?

- a)  $(\frac{7}{2}, 0)$    b)  $(6, 0)$    c)  $(\frac{14}{3}, 0)$    d)  $(\frac{9}{2}, 0)$    e)  $(10, 1)$

13. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work in foot-pounds is done in stretching it 3 feet beyond its natural position?
- a) 600   b) 30   c) 1500   d) 450   e) 100

14. Which of the following integrals represents the volume of the solid generated by revolving the region between  $y = x^2$  and  $y = 2x$  about the  $y$ -axis?
- a)  $\pi \int_0^2 (2x - x^2)^2 dx$    b)  $\pi \int_0^2 4x^2 - x^4 dx$    c)  $2\pi \int_0^2 x(2x - x^2) dx$
- d)  $\pi \int_0^4 (\frac{y}{2} - \sqrt{y})^2 dy$    e)  $\pi \int_0^2 (\frac{y^2}{4} - y) dy$

**Part II. Show enough work. Please box your answer.**

15. (8 points) A farmer has 6000 feet of fencing (posts, wire, whatever), and wishes to inclose a **rectangular** area. One side of the area is bounded by a straight existing fence, so the farmer only needs to add the other three sides. How should the farmer lay out the fence to inclose the maximum area?

16. (7 points) Perform the given integration:  $\int_0^2 x^2(4 + x^3)^{1/2} dx$

17. (7 points) The radius of a circle is given as 10 cm, with a possible error of measurement equal to 1 mm. Use differentials to estimate the maximum error in the area in  $cm^2$ .

18. (7 points) Perform the given integration:  $\int \frac{e^{2x} + 1}{e^x} dx$

19. (8 points) An astronaut stands on a platform 3 meters above the moon's surface and throws a rock directly upward with an initial velocity of 32 m/s. Given that the acceleration due to gravity on the moon's surface is  $1.6 \frac{m}{s^2}$  how high, above the surface of the moon, will the rock travel?

20. (7 points) A student 5 feet tall is 10 feet away from a lamppost 15 feet tall. she is walking away from the lamppost at 2 feet per second. How fast is the tip of her shadow moving away from the foot of the lamppost?