

February 16, 2005

Name: _____

Math 222—EXAM I

Show necessary work!

1. (8 pts) Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{pmatrix}$

(a) $AB =$

(b) $B - 2A^T =$

2. (18 pts) Determine whether the given subset H of the vector space V is a subspace of V .

(a) $V = \mathbb{R}^2$, $H = \{(x, y) | y \geq 0\}$

(b) $V = \mathbb{R}^2$, $H = \{(x, y) | x = y\}$

(c) $V = \mathbb{R}^3$, $H = \{(a, b, 2a - b) | a, b \in \mathbb{R}\}$

(d) $V = P_4$, $H = \{p \in P_4 | p(2) = 0\}$

(e) $V = P_3$, $H = \{a + bx + cx^2 \in P_3 | abc = 0\}$

(f) $V = C[0, 1]$, $H = \{f \in C[0, 1] | f(0) = 2\}$

3. (9 pts) Does the $S = \{1 - x, x^2 + x - 1, x^2 - x - 1\}$ span P_3 ? Give reasons.

4. (8 pts) Solve the system:

$$\begin{aligned}2x_1 + 3x_2 + x_3 &= 1 \\x_1 + x_2 + x_3 &= 3 \\3x_1 + 4x_2 + 2x_3 &= 4\end{aligned}$$

5. (12 pts) Define

(a) Row equivalent matrices

(b) Linear combination

6. (9 pts) Use mathematical induction to prove that if A is a $(n+1) \times (n+1)$ matrix with two identical rows then $\det(A) = 0$.

7. (21 pts) True or False

- (a) ____ If A and B are $n \times n$ nonsingular matrices, then AB is nonsingular and $(AB)^{-1} = A^{-1}B^{-1}$.
- (b) ____ If A is any $n \times n$ matrix, then AA^T is a symmetric matrix.
- (c) ____ The set of all solutions to an $m \times n$ linear system forms a subspace of \mathbb{R}^n .
- (d) ____ A linear system may have exactly two solutions.
- (e) ____ For $n \times n$ nonzero matrices A, B, C if $AC = BC$, then $A = B$.
- (f) ____ The $n \times n$ linear system, represented by the matrix equation $AX = b$ has a unique solution X if and only if A is invertible.
- (g) ____ If A is an $n \times n$ matrix and c is a scalar then $\det(cA) = c^n \det(A)$.

8. (15 pts)

- (a) Use the elimination method to calculate the determinant $\det(A)$ of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 2 & -1 & 0 \end{pmatrix}$$

- (b) Is the matrix A nonsingular? Why?

- (c) What is the cofactor A_{12} for the matrix A above?