

April 17, 2008

Name: _____

Math 222—EXAM III

Show necessary work!

1. (20 pts) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T(x, y) = (2x - y, -x + y)$.
 - (a) Compute the matrix A of T with respect to the standard basis of \mathbb{R}^2 .
 - (b) What is the characteristic polynomial of A ?
 - (c) Is the matrix A diagonalizable? Explain.
 - (d) Compute the matrix B of T with respect to the basis $v_1 = (\frac{1}{2}, \frac{1}{2})$ and $v_2 = (\frac{1}{2}, -\frac{1}{2})$ of \mathbb{R}^2 .
2. (14 pts) Find the standard matrix representation for the linear transformation $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ which doubles the length of each vector \vec{x} and then rotates it by 45° in the clockwise direction.
3. (12 pts) Show that if A and B are similar matrices then $\det(A) = \det(B)$.
4. (12 pts) Let $B = A - \alpha I$ for some scalar α . How do the eigenvalues of A and B compare? Explain.
5. (14 pts) Find the general solution to the following system:

$$\begin{aligned}y_1' &= y_1 - 2y_2 \\y_2' &= -2y_1 + 4y_2\end{aligned}$$

6. (16 pts) Consider the matrix

$$A = \begin{pmatrix} -2 & -1 \\ 6 & 3 \end{pmatrix}$$

- (a) Factor the matrix A into a product XDX^{-1} where D is diagonal.
 - (b) Use this factorization to compute A^6 .
7. (12 pts) Let A be a 3×3 matrix and let λ be an eigenvalue of multiplicity 2. If $A - \lambda I$ has rank 2, is A defective? Explain.